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OF MATHEMATICAL SCIENCE

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THE THIRTY-SIXTH SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY

The thirty-sixth summer meeting and fourteenth colloquium of the Society were held at Brown University, Providence, Rhode Island, from Tuesday to Friday, September 9–12, 1930, preceded by the summer meeting of the Mathematical Association of America.

This was the second meeting of the Society held at Brown University, the first being in the summer of 1914 on the occasion of the celebration of the 150th anniversary of the founding of the institution. Of the fifty members in attendance at that meeting, nineteen were present this year. It is worthy of note that the number of members present sets a record nearly fifty percent higher than that of any previous summer meeting.

The colloquium lectures by Professor Solomon Lefschetz were delivered on Tuesday and Thursday afternoons and Wednesday and Friday mornings with one hundred fifty one members in attendance; these lectures will appear in book form within a few months. By invitation of the Committee on Program, Professors T. H. Hildebrandt and J. D. Tamarkin gave addresses entitled *Linear transformations in general spaces*, and *Distribution of characteristic values of linear integral equations*. Sectional sessions for Analysis were held on Tuesday afternoon and Thursday and Friday mornings; for Geometry, for Foundations, and Point Sets on Tuesday afternoon; for Algebra on Thursday morning; for Applied Mathematics on Friday morning.

During the latter part of Monday afternoon Mr. Norman Isham, Providence architect and a recognized authority on colonial architecture and furniture, led a group of visiting mathematicians and their friends on a walk through the old part of Providence to observe some of the more interesting examples of colonial architecture. On Monday evening President and Mrs. Barbour of Brown University tendered a reception to the guests in Alumnæ Hall. Wednesday afternoon was devoted to an excursion to Newport where the visitors availed themselves of the various opportunities for sight seeing and where a Rhode Island shore dinner was served.

A picture was taken on the steps of Alumnæ Hall on Tuesday, a copy of which is to be inserted later in this Bulletin. In the late afternoon tea was served by the ladies of the Department of Mathematics of Brown University.

The Rhode Island School of Design admitted visiting mathematicians to its museum of art and colonial furniture without fee; and through the courtesy of the University and other local organizations golf and tennis were available to members without charge.

The fine collection in the mathematical department library was open to visitors, who were received by Professor R. C. Archibald.

The dormitories at Pembroke College were open for the visitors and meals were served in the cafeteria. Alumnæ Hall served as a delightful social center. The sessions were held in the Jesse Metcalf Chemical Laboratory and the Arnold Biological Laboratory on the main campus of the University.

The attendance included the following two hundred eighteen members of the Society:

C. R. Adams, R. B. Adams, P. L. Alger, E. L. Anderton, R. C. Archibald, C. L. Bacon, W. D. Baten, A. A. Bennett, Theodore Bennett, A. C. Berry, E. M. Berry, G. D. Birkhoff, H. F. Blichfeldt, G. A. Bliss, M. G. Boyce, Laura Brant, R. W. Brink, A. B. Brown, J. A. Bullard, L. H. Bunyan, W. H. Bussey, S.S. Cairns, W.D. Cairns, B.H. Camp, C.C. Camp, A.D.Campbell, G. A. Campbell, G. N. Carmichael, A. B. Chace, W. F. Cheney, L. P. Copeland, C. H.Currier, H.B. Curry,H.T. Davis, F.F. Decker, L.S. Dederick, Arnold Dresden, W. H. Durfee, L. A. Dye, Allegra Eckles, John Eiesland, H. T. Engstrom, H. P. Evans, D. A. Flanders, W. W. Flexner, A. L. Foster, Philip Franklin, Orrin Frink, T. C. Fry, C. A. Garabedian, H. A. Garabedian, H. M. Gehman, J. J. Gergen, B. P. Gill, D. C. Gillespie, R. E. Gilman, O. E. Glenn, J. W. Glover, M. C. Graustein, W. C. Graustein, B. F. Groat, H. A. Grout, E. S. Hammond, E. R. Hedrick, A. O. Hickson, T. H. Hildebrandt, Einar Hille, T. R. Hollcroft, C. T. Holmes, H. L. Holmes, Aline Huke, E. V. Huntington, H. B. Huntley, W. A. Hurwitz, J. I. Hutchinson, M. H. Ingraham, Dunham Jackson, R. L. Jeffery, R. A. Johnson, F. E. Johnston, F. C. Jonah, Herman Karnow, O. D. Kellogg, A. J. Kempner, Claribel Kendall, L. S. Kennison, M. V. Kenny, T. H. Kiang, B. F. Kimball, S. H. Kimball, F. W. Kokomoor, H. L. Krall, H. W. Kuhn, J. H. Kusner, W. D. Lambert, K. W. Lamson, A. W. Landers, R. E. Langer, Solomon Lefschetz, E. E. Libman, R. B. Lindsay, S. B. Littauer, M. I. Logsdon, C. I. Lubin, J. J. Luck, E. O. McCormick, John McDonnell, L. A. Mac Coll, C. C. MacDuffee, H. P. Manning, Morris Marden, E. N. Martin, A. E. Meder, H. A. Merrill, E. J. Miles, Norman Miller, E. B. Mode, E. C. Molina, C. L. E. Moore, C. N. Moore, L. T. Moore, R. K. Morley, Richard Morris, Marston Morse, David Moscovitz, A. F.

Moursund, F. H. Murray, J. R. Musselman, J. H. Neelley, C. O. Oakley, P. S. Olmstead, H. L. Olson, F. W. Owens, H. B. Owens, F. W. Perkins, H. R. Phalen, James Pierpont, Hillel Poritsky, G. B. Price, S. M. Rambo, W. R. Ransom, Nicholas Rashevsky, S. E. Rasor, A. G. Rau, T. H. Rawles, C. J. Rees, M. S. Rees, C. N. Reynolds, Harris Rice, R. G. D. Richardson, D. E. Richmond, L. V. Robinson, Robin Robinson, C. F. Roos, George Rutledge, S. A. Schelkunoff, C. H. W. Sedgwick, C. E. Seely, Edward Sellers, I. M. Sheffer, W. A. Shewhart, W. G. Simon, L. G. Simons, T. M. Simpson, E. B. Skinner, H. E. Slaught, H. L. Slobin, M. M. Slotnick, C. H. Smiley, A. H. Smith, C. E. Smith, J. J. Smith, W. M. Smith, Virgil Snyder, I. S. Sokolnikoff, Joseph Spear, M. J. Sperry, C. C. Spooner, A. H. Sprague, E. T. Stafford, M. E. Stark, O. H. Stecker, M. E. Steinmetz, M. H. Stone, W. P. Suesman, M. C. Suffa, J. D. Tamarkin, H. B. Thiessen, T. Y. Thomas, M. M. Torrey, J. I. Tracey, W. J. Trjitzinsky, B. M. Turner, Oswald Veblen, C. H. Vehse, G. W. Walker, H. S. Wall, J. L. Walsh, A. C. Washburne, C. W. Watkeys, Warren Weaver, D. W. Weeks, V. H. Wells, R. A. Whelan, H. S. White, Hassler Whitney, Norbert Wiener, E. P. Wiggin, C. E. Wilder, R. L. Wilder, W. L. G. Williams, A. H. Wilson, E. W. Wilson, W. A. Wilson, R. G. Wood, F. S. Woods, F. M. Wright, J. W. Young.

The Secretary announced the election of the following forty-four persons to membership in the Society:

Mr. Howard Holston Alden, Massilon, Ohio;
Professor Hugh R. Beveridge, Monmouth College;
Mr. Walter Edwin Bobertz, Westinghouse Electric Company, Wilkinsburg, Pennsylvania;
Professor Raymond Willis Boydston, Miami University;
Mr. Harry Edward Burton, U. S. Naval Observatory, Washington;
Miss Elizabeth Morgan Cooper, University of Illinois;
Dr. Max Coral, University of Chicago;
Professor Byron Cosby, State Teachers College, Kirksville, Missouri;
Mr. Bolivar Lang Falconer, Marlin, Texas;
Professor Ragnar A. K. Frisch, Yale University;
Professor George Haley, St. Ignatius University;
Mr. Lawrence DeLoss Hampton, Alabama Polytechnic Institute;
Miss Elizabeth Leigh Harris, Southwestern Bell Telephone Company, St. Louis;
Professor Akira Hirayama, Yamagata Women's Normal School;
Professor Warren Gengembre Hubert, College of the City of New York;
Professor James Alexander Hurry, Western State College, Colorado;
Mr. Horace A. Johnson, Los Angeles;
Mr. Maurice Keesing, New York City;
Miss Esther Ober McCormick, Barnard College;
Reverend Paul A. McNally, Georgetown College;
Mr. Edwin W. Miller, University of Michigan;
Mr. Edward Morris, Willits Water Company, Willits, California;
Professor Albert Henry Mowbray, University of California;

Mr. Milton Clarke Nichols, Worcester Electric Light Company, Worcester, Massachusetts;
Professor H. Randolph Pyle, Penn College, Oskaloosa, Iowa;
Professor Calvin Gilbert Reen, Gettysburg College;
Professor Henry Schultz, University of Chicago;
Mr. Jesse W. Shuman, Minneapolis;
Professor Edmond Siroky, Washington University;
Professor Charles H. Smiley, Brown University;
Mr. Paul Szabo, Jr., New York City;
Mr. Henry Bertrand Thiessen, Brown University;
Professor Herbert Anderson Toops, Ohio State University;
Mr. Robert J. Walker, Mt. Lebanon, Pittsburg, Pa.;
Mr. Nicholas Weeks Wells, The Texas Company, Port Arthur, Texas;
Mr. Ronald F. Wick, Agricultural and Mechanical College of Texas;
Mr. Frank Leo Wilmer, Odebolt, Iowa;
Mr. Zarmair Zakarian, New York City;

Nominee of the Department of Mathematics, Brown University: Mr. Harold F. S. Jonah;

Nominee of the Department of Mathematics, University of Wisconsin: Mr. Hugh Lonsdale Turrittin;

Nominees of the University of Michigan: Mr. Dawson Gerald Fulton, Mr. Emanuel Henry Hildebrandt, Mr. Charles Conroy Wagner and Mr. Ehrhardt H. Wagner.

The following members of the London Mathematical Society have accepted membership under the reciprocity agreement:

Professor K. Ananda-Rau, Presidency College, Madras, India;
Professor Edward Foyle Collingwood, Trinity College, Cambridge.

It was announced that Professor E. B. Wilson of the Graduate School of Medicine of Harvard University has accepted the invitation of the Society to give the eighth Josiah Willard Gibbs Lecture in connection with the annual meeting in Cleveland.

The following appointments were announced: to represent the Society at the dedication of the new campus at the University of Rochester, Professor Virgil Snyder; as Committee on Arrangements for the summer meeting of 1931, Professors R. W. Brink (chairman), H. H. Dalaker, M. H. Ingraham, H. L. Rietz and E. B. Stouffer.

Invitations for the summer meeting of 1932 were received from the University of California and from the University of California at Los Angeles. These invitations were accepted and a committee was appointed to work out the details of a

meeting to include both institutions as well as excursions to other neighboring universities. This committee consists of Professors L. D. Ames, E. T. Bell, H. F. Blichfeldt, P. H. Dans and T. M. Putnam (chairman).

The prices of the Bulletin and Transactions beginning with 1931 were fixed at $9.00.

It was announced that for 1930–31 the National Academy of Sciences had allotted to the Society from the funds donated by the General Education Board the sum of $4500, to be distributed as follows: Bulletin, $500; Transactions, $1500; American Journal of Mathematics, $1500; Colloquium Publications, $1000.

The Society agreed to cooperate with the Society for the Promotion of Engineering Education which plans to hold a summer school of two weeks for the teachers of mathematics to engineering students, immediately preceding the summer meeting in Minneapolis, September 7–11, 1931. An advisory committee was appointed consisting of Professor W. D. Cairns, Dr. T. C. Fry, Professors E. R. Hedrick, E. V. Huntington, R. G. D. Richardson and J. W. Young.

A list of nominations for trustees, and officers and other members of the Council was adopted and ordered printed on the ballot.

The weather left nothing to be desired, and the local committee with Professor C. R. Adams as chairman had made careful plans for the arrangements for the meetings.

Resolutions were passed expressing appreciation of the cordial hospitality of the University and of the various other institutions which contributed to the success of the occasion.

The titles and cross references to abstracts of the papers read at the sessions follow below. The papers whose abstract numbers were followed by the letter *t* were read by title; the papers numbered 1 to 6, 22 to 45, 59 to 63, and 72 to 78 were read before the section for Analysis, President Hedrick and Professors Birkhoff, Bliss, and Kellogg presiding; those numbered 7 to 21 were read before the section for Geometry, Foundations, and Point Sets, Professor Graustein presiding; those numbered 46 to 58 before the section for Algebra, Professor Snyder presiding; those numbered 64 to 71 before the section for Applied Mathematics, Professor Weaver presiding; Professor Griffin's paper was read by Professor V. H. Wells. Mr. Cleve-

land was introduced by Professor R. L. Moore, Dr. Féraud and Miss Wilder by Professor Jackson, Mr. Bisshopp by President Hedrick.

1. *Note on the number of linearly independent Dirichlet series that satisfy certain functional equations*, by Professor J. I Hutchinson. (Abstract No. 36–7–311.)

2. *On certain equivalent methods of summation*, by Professor C. N. Moore. (Abstract No. 36–9–317.)

3. *On the summability of Fourier series.* Third note, by Professors Einar Hille and J. D. Tamarkin. (Abstract No. 36–9–318.)

4. *Note on the convergence of a sequence of approximating polynomials*, by Professor Dunham Jackson. (Abstract No. 36–9–319.)

5. *An application and an extension of Pierpont's theory of integration*, by Professor R. L. Jeffery. (Abstract No. 36–9–320.)

6. *As to a "Goldschmidt solution" for a wire of minimum attraction*, by Professor F. L. Griffin. (Abstract No. 36–9–321.)

7. *Pencils of hypersurfaces*, by Professor T. R. Hollcroft. (Abstract No. 36–9–322.)

8. *Closed circuits upon polyhedra*, by Professor C. N. Reynolds. (Abstract No. 36–7–316.)

9. *On a certain class of ruled* V_{n-1}^{n-1} *in* S_n. Second paper, by Professor John Eiesland. (Abstract No. 36–7–309.)

10. *An axiomatic basis for euclidean plane geometry*, by Dr. S. S. Cairns. (Abstract No. 36–9–323.)

11. *Functionality in combinatory logic*, by Professor H. B. Curry. (Abstract No. 36–9–324-*t*.)

12. *A converse of the theorem regarding the separation of* E_3 *by a closed two-dimensional manifold of genus p*, by Professor R. L. Wilder. (Abstract No. 36–3–196.)

13. *On the regular points of a continuum*, by Professor W. L. Ayres. (Abstract No. 36–7–307-*t*.)

14. *Element transformations and isogonal series*, by Professor Edward Kasner. (Abstract No. 36–7–313–*t*.)

15. *Concerning the possibility of a certain binary quartic being a line section of the plane quartic curve of genus zero*, by Professor J. H. Neelley. (Abstract No. 36–9–325–*t*.)

16. *A continuous development of mathematical logic in finite terms*, by Mr. A. L. Foster. (Abstract No. 36–9–326–*t*.)

17. *A property of continua similar to local connectivity*, by Professor W. A. Wilson. (Abstract No. 36–9–327–*t*.)

18. *Semi-metric spaces*, by Professor W. A. Wilson. (Abstract No. 36–9–328–*t*.)

19. *Concerning continuous curves without local separating points*, by Professor G. T. Whyburn. (Abstract No. 36–9–329–*t*.)

20. *Concerning the subsets of regular curves*, by Professor G. T. Whyburn. (Abstract No. 36–9–330–*t*.)

21. *On the existence of acyclic curves satisfying certain conditions with respect to a given continuous curve*, by Mr. C. M. Cleveland. (Abstract No. 36–9–331–*t*.)

22. *Concerning a certain integral equation*, by Dr. L. H. Bunyan. (Abstract No. 36–9–332.)

23. *On certain differential equations*, by Dr. J. A. Shohat. (Abstract No. 36–9–333–*t*.)

24. *The inversion of a differential operator of infinite order of Fuchsian type*, by Professor H. T. Davis. (Abstract No. 36–9–334.)

25. *The problems of Lagrange and Mayer under general end conditions*, by Professor Marston Morse. (Abstract No. 36–9–335.)

26. *Prime number theory and Planck's law*, by Professor Norbert Wiener. (Abstract No. 36–9–336.)

27. *Preliminary report on a new method in the theory of central orbits*, by Professor O. E. Glenn. (Abstract No. 36–7–310.)

28. *A matrix theory of measurement*, by Professor A. H. Copeland. (Abstract No. 36–9–337–*t*.)

29. *Summation factors which are powers of a complex variable*, by Professor W. H. Durfee. (Abstract No. 36–9–338.)

30. *Some properties of the oscillation of harmonic functions*, by Dr. F. W. Perkins. (Abstract No. 36–9–339.)

31. *The generalized Euler differential equation of infinite order*, by Professor H. T. Davis. (Abstract No. 36–9–340–*t*.)

32. *On Stieltjes' differential equation*, by Dr. Morris Marden. (Abstract No. 36–9–341–*t*.)

33. *Note on the methods of Sturm*, by Professor C. O. Oakley. (Abstract No. 36–9–342–*t*.)

34. *On interlinkings*, by Professor G. Y. Rainich. (Abstract No. 36–7–315–*t*.)

35. *Note on the overconvergence of sequences of polynomials of best approximation*, by Professor J. L. Walsh. (Abstract No. 36–9–343–*t*.)

36. *The approximation of harmonic functions in three dimensions by harmonic polynomials*, by Professor C. T. Holmes. (Abstract No. 36–9–344–*t*.)

37. *Poincaré's rotation number and Morse's type number*, by Dr. G. A. Hedlund. (Abstract No. 36–9–345–*t*).

38. *Geodesics on a two-dimensional Riemannian manifold with periodic coefficients*, by Dr. G. A. Hedlund. (Abstract No. 36–9–346–*t*.)

39. *On Birkhoff's Pfaffian systems*, by Dr. Lucien Féraud. (Abstract No. 36–9–347–*t*.)

40. *On the symmetric form of the Fourier sine and cosine transforms*, by Dr. S. B. Littauer (National Research Fellow). (Abstract No. 36–9–348–*t*.)

41. *The effects of general regular transformations on oscillations of sequences of functions*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 36–9–349–*t*.)

42. *Note on the existence of a positive function orthogonal to a given set of functions*, by Dr. N. H. McCoy (National Research Fellow). (Abstract No. 36–9–350–*t*.)

43. *Theorems concerning functional geodesic coordinates*, by Professor A. D. Michal. (Abstract No. 36–7–303–*t*.)

44. *Scalar functional invariants of a continuous one-parameter family of functional vectors*, by Professor A. D. Michal. (Abstract No. 36–7–304–*t*.)

45. *An introduction to differential geometries of composite spaces*. Preliminary report, by Professor A. D. Michal. (Abstract No. 36–7–305–*t*.)

46. *An elementary theorem on matrices*, by Professor M. H. Ingraham. (Abstract No. 36–7–312.)

47. *The construction of a headless and groupless triad system on 31 elements*, by Professor H. S. White. (Abstract No. 36–9–351.)

48. *Residue systems for a prime modulus*, by Professor A. J. Kempner. (Abstract No. 36–9–352.)

49. *On sequences defined by linear recurrence relations*, by Dr. H. T. Engstrom (National Research Fellow). (Abstract No. 36–9–353.)

50. *Algebraic Pfaffians*, by Professors C. L. E. Moore and Philip Franklin. (Abstract No. 36–9–354.)

51. *Expansions of arithmetical functions in infinite series*, by Professor R. D. Carmichael. (Abstract No. 36–9–355–*t*.)

52. *On the representation of integers as sums of an even number of squares or of triangular numbers*, by Professor R. D. Carmichael. (Abstract No. 36–9–356–*t*.)

53. *Concerning quasi-k-fold transitivity of permutation groups*, by Professor R. D. Carmichael. (Abstract No. 36–9–357–*t*.)

54. *On a question related to Waring's problem*, by Professor R. D. Carmichael. (Abstract No. 36–9–358–*t*.)

55. *Abstract defining relations for the simple group of order* 5616, by Mr. K. E. Bisshopp. (Abstract No. 36–7–308–*t*.)

56. *The characteristic roots of a matrix*, by Professor E. T. Browne. (Abstract No. 36–9–359–*t*.)

57. *On the separation property of the roots of the secular equation*, by Professor E. T. Browne. (Abstract No. 36–9–360–*t*.)

58. *On some general commutation formulas*, by Dr. N. H. McCoy (National Research Fellow). (Abstract No. 36–7–314–*t*.)

59. *Projective transformations in function space*, by Dr. L. S. Kennison. (Abstract No. 36–7–306.)

60. *Semi-linear integral equations*, by Professor C. O. Oakley. (Abstract No. 36–9–361.)

61. *Convergence and summability criteria for Fourier series*, by Dr. J. J. Gergen (National Research Fellow). (Abstract No. 36–9–362.)

62. *A useful formula in the theory of finite differences*, by Professor B. F. Kimball. (Abstract No. 36–9–363.)

63. *A rule of signs involving certain orthogonal polynomials*, by Dr. Morris Marden. (Abstract No. 36–9–364.)

64. *A study of the interference of polarized light by the method of coherency matrices*, by Dr. Dorothy W. Weeks. (Abstract No. 36–9–365.)

65. *On the flow of heat for a solid in contact with a liquid*, by Professor R. E. Langer. (Abstract No. 36–9–366.)

66. *Some differential and algebraic consequences of the Einstein field equations*, by Professor K. W. Lamson. (Abstract No. 36–9–367.)

67. *Some new mathematical problems in biophysics*, by Mr. Nicholas Rashevsky. (Abstract No. 36–9–368.)

68. *Mean value of the ordinate of the locus of the rational integral algebraic function of degree n, expressed as a weighted mean of n+1 ordinates, and the resulting rules of quadrature*, by Mr. B. F. Groat. (Abstract No. 36–9–369.)

69. *Corrections for the moments of a frequency distribution in two variables*, by Dr. W. D. Baten. (Abstract No. 36–9–370.)

70. *Moderately thick circular plates with plane faces*, by Professor H. W. Sibert. (Abstract No. 36–9–371–*t*.)

71. *Correlation coefficients and transformation of axes*, by Miss Marian A. Wilder. (Abstract No. 36–9–372–*t*.)

72. *Linear transformations in conjugate abstract spaces*, by Dr. A. C. Berry (National Research Fellow). (Abstract No. 36–9–373–*t*.)

73. *On conservative transformations connected with certain harmonic functions*, by Dr. Hillel Poritsky. (Abstract No. 36–9–374–*t*.)

74. *Problems of closest approximation connected with the solution of linear differential equations*, by Professor W. H. McEwen. (Abstract No. 36–9–375–*t*.)

75. *Capacity of sets of Cantor type*, by Professor O. D. Kellogg. (Abstract No. 36–9–376–*t*.)

76. *On the Padé approximants associated with certain convergent series*, by Professor H. S. Wall. (Abstract No. 36–9–377.)

77. *A further note on singular points of the differential equation* $dx/X(x, y) = dy/Y(x, y)$, by Dr. C. I. Lubin. (Abstract No. 36–9–378–*t*.)

78. *A solution of a problem of Birkhoff*, by Dr. W. J. Trjitzinsky (National Research Fellow). (Abstract No. 36–9–379.)

R. G. D. RICHARDSON,
Secretary

DEDEKIND'S WORKS, VOLUME I

Richard Dedekind. Gesammelte mathematische Werke. Herausgegeben von Robert Fricke, Emmy Noether, Öystein Ore. Erster Band. Braunschweig, Friedr. Vieweg & Sohn, 1930. 397 pp.

Richard Dedekind's 85 years (1831–1916) spanned one of the most fruitful periods in the history of mathematics. His dissertation is dated at Göttingen in 1852, about two years before the death of Gauss, and he lived to see Einstein's theory of general relativity and gravitation. The peer of his great contemporaries, Dedekind was probably less widely known than any of them. Those whose inclinations led them to inspect his more abstract creations recognized the greatness of the work, but by the general public, even among leading mathematicians, the most characteristic products of Dedekind's genius were neglected. For it was and still is possible to be a highly competent mathematician without ever having heard of the theory of ideals. This needs but slight qualification even when account is taken of the infiltration of modern arithmetical concepts into the theory of algebraic functions and other branches of mathematics; the ideas of the theory, but scarcely the theory itself, are achieving a certain measure of popularity for what it may be worth.

Dedekind's material career seems to have been strangely like his masterpiece; he never occupied one of the great chairs in his native country or elsewhere. And it would be difficult to find anywhere a more superb illustration of the usual idiotic logic of history than in the rise of Dedekind's popular scientific reputation. For one mathematician who knows his way about in the Dedekind theory of ideals, there are probably hundreds who have been familiar with the Dedekind cut from their youth up. The thing is notorious; every tyro is indoctrinated with it in his advanced calculus. And now, according to the customary irony of time, this cut, for which Dedekind is hundreds-to-one more famous than for his work in algebraic numbers, seems to be turning into something like a very bad pun, for it is a schism in the official faiths of analysis and a prolific source of interminable polemics. Dedekind himself is said to have hesitated nearly two years before committing it to print.

This first volume of his collected works contains 19 papers in chronological order from 1852 to 1882. According to the editorial plan, the memoirs shall occupy the first two volumes, while the two essays on number (as they are known in America) and the expository French account of the theory of algebraic numbers (Darboux' Bulletin) are reserved for the third volume. This will no doubt appeal to most as a wiser arrangement than a strict adherence to chronological order. The essential parts of the famous supplements on general arithmetic in the second, third and fourth editions of Dirichlet's *Zahlentheorie* are to be reproduced. A life of Dedekind is also promised for the third volume. The printing is beautifully done on good, restful paper.

The editing of the present volume has been carried through with unusual care, and with some features of marked interest not often found in collected mathematical works. At the end of each of the more fundamental papers, the responsible editor has added short, helpful notes. These are roughly of three kinds: brief historical indications relating Dedekind's work to that of others; incisive criticism—in the reputable, relevant sense; references to important

extensions and completions of Dedekind's work by later writers. The arithmetical papers have been edited by Noether and Ore, those on elliptic modular functions by Fricke. Touching the last topic, Fricke calls attention to the much earlier use by Gauss of the "Dreiecksnetz." There does not yet seem to have been discovered a similar anticipation by Gauss of the theory of ideals, either Kummer's or Dedekind's. This is a little surprising, as Gauss could hardly have stopped short—by luck or lack of curiosity—with those fields in which the fundamental theorem of arithmetic happens to hold. Where necessary, the editors have added footnotes, of the same general character as the others.

Whether or not Dedekind's theory of ideals is to be the final one, many believe that it will endure as one of the great landmarks in mathematics. Indeed, the work of Ore has cast a new light on the whole field, showing that by an easier approach the desired end is attainable by strictly finite means. Whatever may be the degree of resemblance between the ultimate theory of algebraic numbers and Dedekind's, the creation of Dedekind will no more be abolished by the new than was Euclid by Riemann or relativity. The fruitfulness of Dedekind's theory is becoming daily more evident.

When a step of the historical importance of Dedekind's theory has been taken, its interest and its influence abide. And it is of some significance to have the history of a such a step exact. In this respect there still seems to be a trifle of haziness, even after the painstaking work of the editors. Without the final theorem of Dedekind concerning certain troublesome divisors, the theory was incomplete; in fact it was not a theory, for without the final theorem it failed to solve completely the problem proposed. It would be of some interest to settle once for all the date of first publication of the complete theory of Zolotareff. The unsatisfactory first draft was the only one known to Dedekind —after he had elaborated his theory; the complete theory of Zolotareff until comparatively recently seems to have escaped notice. Dedekind and Zolotareff worked in total independence of one another. Zolotareff committed suicide so early (a girl, not his theory) that he had no opportunity to exploit his discoveries. That elusive date has a certain human interest as well as historical.

After renewing acquaintance with some of these great papers, and being stirred again by their sublimity of conception, one is slightly relieved to find that Dedekind after all was human. One short note in this volume tells the story. It concerns a trifle about quadratic residues which had attracted Sylvester's gadding attention. From the tone of his remarks about the "Editors of Dirichlet's works"—the *Zahlentheorie*—it would seem that Sylvester was unaware at the time of Dedekind's existence. At any rate he trod on the remiss Editor's toes in truly Olympian fashion. Dedekind, of course, was in the right, until he allowed himself the doubtful luxury of that most human lapse of good judgment, a tart reply in print to a bumptious critic.

Dedekind's style is restrained, at times almost cold. But he put all that was in him into his work, and that all was tremendous. His admirers will hope for a revealing portrait of this great genius in the third volume, which will show him to be something more than a legend and a name.

E. T. Bell

SHORTER NOTICES

Zur Geschichte der Logik. By Federigo Enriques (L. Bieberbach, Translator). (Vol. 26 of Wissenschaft und Hypothese.) Leipzig and Berlin, B. G. Teubner, 1927. 240 pp.

This small book bears the subtitle "Grundlagen und Aufbau der Wissenschaft im Urteil der mathematischen Denker." It is a narrative history of logic with some emphasis upon the logical problems of mathematics. Although the "Namenverzeichnis" lists references to some three hundred persons cited in the book, the treatment refers largely to periods and movements. The work is divided into four parts: I. The logic of the ancients, II. Rationalism and the development of modern logic (extending from Bacon through Kant), III. The modern reform of logic, IV. Supplement. From inductive logic to the logic of scientific systems.

Part I, which covers only 42 pages, is a well-digested summary of the logical views of the Greeks, supplemented by brief remarks on medieval logic. Plato, Aristotle, Euclid, the Stoics, the Sceptics are all discussed. In a similar manner Part II discusses modern developments prior to the reform in the Nineteenth Century. While Leibniz, Descartes, Newton are all mentioned, the mathematical features are treated as incidental to the larger movements. Part III summarizes the contributions of the nineteenth century and early twentieth century toward the problem of the significance of postulational systems, starting with non-euclidean geometry. The discussion is singularly clear, inclusive and impartial. This is the section of the book of special mathematical interest. It is no longer novel, despite the continued medieval flavor of many American texts on plane geometry. Part IV brings the book to a natural conclusion by discussing the role of logic in modern natural science, with mention of Einstein, Eddington and others.

The work is one of general scientific interest, available to any reader, but of added significance in proportion as one has read the basic philosophical writings to which references are made. No technical knowledge (except incidentally of the Latin and Greek languages) is presupposed. The treatment is singularly well-knit and bold summarizing remarks are frequent. Present controversial material is strikingly absent. This makes the book safe and informative but the reader may be disappointed in failing to find more than hints of those very topics which are today arousing interest among mathematicians in questions of logical import. The work in its breadth of view and literary style maintains Enriques as a master of this field of exposition.

A. A. BENNETT

Praktische Infinitesimalrechnung. By F. F. P. Bisacre. Berechtigte Deutsche Ausgabe unter Mitwirkung von E. Trefftz, herausgegeben von E. König. Leipzig, Teubner, 1929. 12+364 pp.

Bisacre's *Applied Calculus* was published in 1921 and was reviewed in this Bulletin, volume 28 (1922), page 471. For the German translation the text has been carefully revised but no essential changes have been made.

W. R. LONGLEY

Collected Geometrical Papers. Part I. By Syamadas Mukhopadhyaya. Calcutta, University Press, 1930. 159 pp.

The present collection contains 17 papers, previously published for the most part in Asiatic periodicals in the interval 1908–1928.

The topics considered fall into three general heads, those concerning topological questions, including cyclic points, sextactic points, etc. of plane curves, those concerning triangles and quadrilaterals in hyperbolic geometry, and those on methods of visualizing representations of four way space.

The methods are mostly those of elementary geometry, although differential expressions are used freely in the treatment of osculation. The proofs are strikingly direct and simple, and many of the theorems were first published previously to those obtained by others. For workers in topology, the Papers will be of real service.

VIRGIL SNYDER

Theory and Application of Infinite Series. By K. Knopp. Translated from the second German edition by R. C. Young. London, Blackie, 1928. 12+571 pp.

The first edition of Professor Knopp's book on infinite series appeared in 1922 and was reviewed in this Bulletin, volume 29 (1923), pp. 474–475. Two years later a second edition was necessary and the author took occasion to improve the details of expression and demonstration throughout and to rewrite the last chapter, dealing with divergent series. The clear and elegant presentation has made this treatise a standard introduction to the subject and English readers are fortunate in having now a skillfully prepared translation. In addition to the material contained in the second edition the author has added a chapter on Euler's summation formula and asymptotic expansions, a subject which he omitted with great reluctance from the German edition. The book is designed particularly for those who may wish to acquire a knowledge of the subject by private study and contains a large collection of exercises.

W. R. LONGLEY

Über die Verhältniszahl des Goldenen Schnitts, die Reihe der mit ihr zusammen hängenden ganzen Zahlen und eine aus dieser abgeleitete Reihe. By Ludwig Kaiser. Berlin, Teubner, 1929. 123 pp.

The popular accounts of the Golden Section mostly delight in its mysterious connections with the shape of picture-frames, matchboxes, the Great Pyramids, cathedrals, the distribution of leaves on herbs and trees, etc., etc. In the present elementary book the author only proposes to derive the arithmetical properties of the Golden Section and related numbers. The book is clearly written and probably intended for interested pupils in the German "Gymnasium." Part I deals with various properties of $g=(1+\sqrt{5})/2$ and its powers, most of them easily obtainable from the well known theorems on the algebraic field $k(\sqrt{5})$ and continued fractions. Part II contains an investigation of the arithmetic properties of the numbers in the series of Kepler (Lambert), which is the same as the series of denominators or numerators of the successive convergents in the continued fraction for g.

OYSTEIN ORE

Die Lehre von den Kettenbrüchen. By O. Perron. Second edition, revised. Leipzig and Berlin, Teubner, 1929. xii+524 pp.

This second edition is still the most exhaustive and modern of all expositions of the theory of continued fractions. It is mainly a photomechanic reproduction of the first edition (1913), except paragraph 56, where some theorems on a type of almost periodic continued fractions have been extended considerably, and chapter 9, which deals with the continued fractions of Stieltjes. In this chapter, nearly all the proofs of the first edition have been greatly simplified, and a brief but very readable account of recent investigations of the moment problem has been added.

T. H. GRONWALL

Mathematik und Logik. By H. Behmann. (Vol. 71 of Mathematisch-Physikalische Bibliothek.) Leipzig and Berlin, Teubner, 1927. 59 pp.

This is a small text-book for elementary students of mathematics presenting in good pedagogical sequence an introduction to a calculus of logic. No deep philosophical speculations are offered, and no unsolved problems are suggested. Not only is illustrative material abundant but the pamphlet closes with a collection of exercises by which the reader may test his proficiency. The discussion is largely formal but is scholarly and is made concretely interesting. After preliminary remarks on the principles of negation, conjunction, distribution, etc., separate sections are devoted to the logic of classes, and the logic of correspondences. Finally the notions are applied to the concept of finite cardinal number, thus introducing the reader not only to the mathematics of logic but also to the logic of mathematics. The notation appears more clumsy and less adequate than that current in this country among the pupils of E. H. Moore, but this is a minor matter. It is just books of this sort that will develop among youthful readers a much-needed interest in the subject.

A. A. BENNETT

Problèmes de Statique Graphique et de Résistance des Matériaux. By L. Roy. Paris, Gauthier-Villars, 1929. 8+119 pp.

Compléments et Exercices sur la Mécanique des Solides. By G. Bouligand, with the collaboration of J. Dollon. Paris, Vuibert, 1929. 8+132 pp.

The first of these two little volumes has been prepared by Professor Roy for the use of his students in the Electro-Technical Institute and in applied mechanics at the University of Toulouse. It consists of a collection of problems and their solutions covering the ground of the second volume of the author's *Cours de Mécanique Appliquée.* The problems follow the order of the text, to which references for the theory are made.

The book by Professor Bouligand is a supplement to the first volume of his *Précis de Mécanique Rationnelle.* The first part, of sixteen pages, is devoted to the kinematics of solid bodies and consists of a brief chapter on the theory followed by a collection of exercises with their solutions.

In the second part we find a detailed consideration of problems involving friction. A treatment of the general theory is followed by chapters on impact with sliding friction and on unilateral constrained motion.

W. R. LONGLEY

Einführung in die Mathematische Behandlung Naturwissenschaftlicher Fragen. Erster Teil: Funktion und Graphische Darstellung. Differential- und Integralrechnung. By A. Walther. Berlin, Springer, 1928. 8+220 pp.

This introduction to the mathematical treatment of questions in natural science is intended primarily for physicists, chemists, and biologists. A detailed review appeared in The American Mathematical Monthly for March, 1930, pages 145–147.

W. R. LONGLEY

Dynamics. By A. S. Ramsey. Cambridge, University Press, 1929. 12+259 pp.

Professor Ramsey, President of Magdalene College, Cambridge, has added one more to the long list of good English texts on mechanics. It is based upon lectures given during many years to students preparing for the Mathematical Tripos and it is assumed that the reader has already acquired some knowledge of elementary dynamics. It is stated in the preface that "an attempt has been made to preserve the conciseness of lecture notes and at the same time to give detailed explanations where experience has shown that students find difficulties." In this attempt the author has succeeded admirably and has included a considerable number of worked examples. But no student can learn much about mechanics without solving problems for himself and the chief value of the book as a teaching text lies in the extensive collection of examples for solution, which have been taken largely from Scholarship and Tripos examination papers.

After three brief chapters on kinematics, the treatment of kinetics is based squarely on Newton's Laws of Motion. Admitting that from a philosophical standpoint Newton's definitions and laws offer much scope for criticism, the author decides to leave to the reader his primary conceptions of time and space and to adopt a Newtonian basis for the development of the subject. "Newtonian Mechanics is the basis of all theoretical work in applied mechanics or engineering and the results of the theory have been and are still being confirmed every day by numerous appeals to experiment, so that nowadays no one questions whether the theory is adequate to furnish reliable results in common matters. It is only when on the one hand we begin to investigate what is relatively very small, e.g., the interior of an atom, or on the other hand when we leave the earth and apply the Newtonian theory on a much wider scale that discrepancies can be detected, and even here it may be remarked that Newtonian Mechanics has proved adequate to enable astronomers to predict the time of eclipses with an accuracy that would hardly have been possible had the foundations of dynamics been radically at fault."

Later, in discussing the motion of particles under the inverse square law, the differential equation of the orbit, as modified by Einstein's Law of Gravitation, is given and the solution shows how this may account for the observed advance of the perihelion of Mercury.

The scope of the book is that of the usual course in dynamics for the better undergraduates who are interested in the subject from the mathematical point of view.

W. R. LONGLEY

Les Problèmes des Isopérimètres et des Isépiphanes. By T. Bonnesen. Paris, Gauthier-Villars, 1929. iv+175 pp.

Here is the latest volume of the Collection Borel. Readers having as little Greek as the reviewer will appreciate the kindness of the author in translating the last word of the title on line 4 of the introduction; it means solids of given surface.

After introductory chapters on elementary maxima and minima, and Jensen's theory of convex functions, the author passes to the main subject matter, viz. the classical extremal properties of the circle and the sphere, and their generalizations, notably the Minkowski inequalities for the mixed volume of three convex solids. The very elegant methods of treatment are original with the author and consist in geometrical constructions, often surprisingly simple, on polygons and polyhedra, supplemented by elementary algebraic calculations and limit operations. The main difficulty in this subject consists in determining the cases where an inequality of the type considered reduces to an equality. This difficulty the author overcomes very neatly by setting up stronger inequalities, from which the answer may be read off at once. For instance, he replaces the ordinary isoperimetric inequality in the plane, $L^2/(4\pi)-S\geqq 0$, by $L^2/(4\pi)-S\geqq(\pi/4)(R-r)^2$, where L is the length of a convex curve, S the area it encloses, and R and r the radii of the circumscribed and inscribed circles. Now it is seen at once that in the ordinary inequality, equality takes place only when $R=r$, that is, for the circle.

The last chapter deals with the extension of the methods used to certain more general problems in the calculus of variations. There is also a fairly complete bibliography.

T. H. Gronwall

Géometrie sur les Surfaces et les Variétés Algébriques. By S. Lefschetz. Mémorial des Sciences Mathématiques, fascicule XL. Paris, Gauthier-Villars, 1929. 66 pp.

This pamphlet is a fairly complete account of that advanced part of the theory of algebraic surfaces and varieties which deals with the transcendental and geometric properties of a variety which are invariant under birational transformations. The timeliness of such a report is unquestionable. Since the appearance of Enriques' and Castelnuovo's article in the Encyclopedia (1914) the theory has been enriched by many new important results, and also a new development of the theory along the lines of analysis situs has taken place (Lefschetz). This development has thrown new light upon the stupendous progress already achieved by the theory thanks to the investigations of the Italian geometers Castelnuovo, Enriques, Severi, etc., and the French analysts Humbert, Picard, Poincaré. The pamphlet thus serves a useful purpose in furnishing a survey of the present state of the theory, and also gives the exact measure of the distance travelled by the theory since 1914.

The exposition centers around three main topics: the transcendental theory of the integrals attached to an algebraic variety V, the geometric theory of linear and continuous systems on V, and the analysis situs of V. Each of these topics leads in its own way to the very heart of the theory of algebraic varieties. The material is so arranged as to bring forth the far reaching con-

nections between these three topics, which thus appear only as different aspects of what one may call the theory of (numerical and functional) invariants of an algebraic variety. Although the geometric, analytic and transcendental points of view are freely allowed to have full play, the stress is laid upon the analysis situs applications, which, we think, is fully justified by the fact that even specialists in algebraic geometry are not yet sufficiently familiar with the topological aspects and methods of the theory.

The author does not confine himself to mere exposition of known results. When necessary, unsolved questions are raised and freely discussed, and this valuable feature of the pamphlet makes it something more than a résumé At the end a rather inspiring list of possible new researches to be pursued is given.

Some details may be criticized. For instance, Theorem 27 on page 24 is given somewhat casually, without any accompanying comment, which is against the historic and intrinsic importance which this theorem had in the development of the theory of algebraic surfaces. In the discussion of the problem of the existence of double integrals of the 1st kind without periods (p. 28) it would have been advisable to mention explicitly the closely connected problem of the existence of simple integrals of the so-called semiexact differentials of the first kind, introduced by Severi.

There are a few misprints which may mislead the reader. On p. 15, XIV, "$k<d-2$," "Γ_{k-2}" and in formula (10) "C^{k+1}" should be respectively: "$k\leqq d-2$," "Γ_k" and "C^{d-k-1}". On p. 16 fourth line from bottom and on p. 17, XX, "C^{d-k}" and "C^{d-k+1}" should be "C^k" and "C^{k+1}", respectively.

OSCAR ZARISKI

Cours de Mécanique Professé à l'École Supérieure des Mines. By P. Lévy Paris, Gauthier-Villars, 1928. 8+205 pp.

This book was prepared by Professor Lévy, of l'École Polytechnique, on the occasion of his assignment to l'École Supérieure des Mines. The point of view differs from that of such standard treatises as Appell's in that more attention is given to mechanical and geometrical considerations. For example in studying the dynamics of a particle, the author feels that it is more important for the student to understand and appreciate the form of the trajectory than the analytic formulas defining it.

The technical applications are almost entirely confined to kinematics. After the first chapter on theoretical kinematics there are two comparatively long chapters on applications to machinery. Then follow eight chapters giving the customary treatment of the dynamics of a particle and of rigid bodies. The next chapter considers the equilibrium of flexible systems and includes a breaf treatment of graphical statics.

It is implied in the preface that this volume is intended to furnish a foundation for the oral instruction of the course and hence it does not contain exercises for the student. Some material not covered in the oral instruction is included in the book with the hope that the curiosity of the better students will be aroused. For this reason the volume closes with a supplementary chapter on the theory of relativity.

W. R. LONGLEY

Leopold Kronecker's Werke, Volume IV. Edited by K. Hensel. Leipzig, Teubner, 1929. 4+509 pp.

It is now more than 30 years since the first three volumes of Kronecker's collected mathematical papers appeared, and the present volume therefore comes as a pleasant surprise. Still more pleasing is the announcement of the editor that the two remaining volumes, Volume V and Volume III², will follow in a short time.

Volume IV contains Kronecker's purely algebraic papers, his investigations on solvable equations and the generation of Abelian fields. Furthermore it contains the first part of his memoirs on the theory of elliptic functions, particularly the real and complex multiplication and the connection with the theory of quadratic forms. At the end of the volume, Professor Hensel has added a set of valuable critical and historical notes.

OYSTEIN ORE

Éléments de Mathématiques Financières. By R. Thiry. Paris, Vuibert, 1930. 87 pp.

While acknowledging the need for the usual text to include details in regard to applications, schedules, etc.,—especially for the student who desires actually to master the various processes—the author seeks to reduce the treatment to a minimum by including only the essential processes. He is primarily interested in those individuals—possibly in adminstrative positions—who desire a general knowledge of those processes. The little book is in nowise a profound treatise and even departs, wittingly or unwittingly, from the standard—or at least widely accepted—forms of notation. The tables are mere extracts. It is doubtful whether most American texts would not remain just as clear and be just as brief if the material omitted in this text were omitted in them also. A small appendix affords a brief review of important but elementary parts of algebra.

C. H. FORSYTH

Leçons de Mécanique Rationnelle. By F. Bouny. Volume 2. Paris, Blanchard, 1929. 8+659 pp.

Professor Bouny's treatise on mechanics is based upon lectures which he has given to students of mining and metallurgy at Mons. The second volume treats the dynamics of a particle and of rigid bodies and presents a good detailed exposition of classical mechanics. The methods are, in general, also classical, although the author uses vector calculus where convenient and there is a brief mention of the use of integral equations in connection with the problem of a vibrating string and resonance.

Because of its present importance the book contains a very extensive study of vibratory motion and an attempt is made to put the results in forms convenient for applications.

A valuable feature is the collection of exercises at the end of each chapter. While a few are merely numerical applications, the great majority serve to amplify the theory. Answers to the exercises are given and in many cases there is also an indication of the method of solution.

W. R. LONGLEY

Grundzüge der Differential- und Integralrechnung. By G. Kowalewski. 4th ed. Leipzig and Berlin, B. G. Teubner, 1928. v+417 pp. Rm 16.

The first edition of this well known text, which appeared in 1909, was reviewed in this Bulletin, vol. 19 (1913), p. 531. The second edition (1919) and the third, which followed soon after, were reprints of the first, with misprints corrected. The same remark holds for the present fourth edition, except that there is a new appendix of eleven pages on Fredholm determinants and integral equations. Here the Fredholm determinants and minors are exhibited as the result of a limit process, their convergence is proved, and (in a page and a half) they are employed in the solution of the linear integral equation of the second kind.

This is a text for the mature student; it will be useful for reference in our courses on functions of real variables rather than in calculus classes. In his introduction the author says, "Ich habe mich bemüht überall möglichst streng zu sein." The standard of rigor is indeed high; one reviewer has complained of the dry brevity that seems to him almost "cold and heartless." Nevertheless, there is true art here.

Even in this fourth edition some annoying misprints remain, but fortunately not many have survived.

D. R. CURTISS

L'Ancienne et la Nouvelle Théorie des Quanta. (Cours de Physique Théorique de la Faculté des Sciences de Paris.) By Eugène Bloch. Paris, Hermann, 1930. 417 pp. 90 fr.

This book is essentially an account of two series of lectures, given at the Sorbonne during 1926–27 and 1928–29. It gives students of the subject an exposition of the historical development of the theory of quanta, and illustrates this by a number of typical examples. It therefore does not claim any other originality than the selection of the material, in which it has very well succeeded.

After a historical introduction comes the Planck radiation law (derived in the way of Einstein, Physikalische Zeitschrift, 1917) and the photoelectric effect, which leads up to the Compton effect. For this a whole chapter is reserved. Then follow the periodical system of the elements, the Bohr theory of the hydrogen atom, and the Sommerfeld quanta conditions with the fine structure. The next chapters deal with atomic magnetism, the Stern-Gerlach experiments, and Goudsmit-Uhlenbeck's theory of the spinning electron. The introductory chapters end with the Zeeman effect and take 195 pages of the book.

The last ten chapters deal with modern quantum theory. A first chapter gives some fundamental results of analytical mechanics. Then comes the Bohr correspondence principle. In Chapter 13, page 244, we find a discussion of the de Broglie waves, in which the Davisson-Germer, G. T. Thomson, Rupp, etc., experiments find their place. The next chapter deals with the Schrödinger equation. Then follow applications to rotator and hydrogen atom, and the statistical interpretation of the Schrödinger function. Chapters 17 and 18, page 321, deal with matrix mechanics. Then comes the principle of indetermination and a last chapter discusses the new statistical mechanics (Bose,

Einstein and Fermi). An appendix explains Legendre, Laplace, Hermite, and Laguerre functions.

Bloch's book seems to be eminently fit for a general introduction to the present state of things. It can be compared to Birtwistle's books (*The Quantum Theory of the Atom*, Cambridge, 1926, *The New Quantum Mechanics*, Cambridge, 1928), but is more recent and is, perhaps, more readable.

D. J. STRUIK

The Adjustment of Errors in Practical Science. By R. W. M. Gibbs. The Oxford University Press, 1929. 112 pp.

The object of this little book is to simplify and condense into a readable form the gist of the theory of errors arising in practical work. Ideas, important formulas and curves of reference basic to this phase of mathematics are presented in a concise, clear and interesting manner. The main body of the treatise is devoted to the presentation of these with diagrams, examples and tables; while the appendix is devoted to the mathematical derivation of the formulas and useful integrals. Students not interested in the mathematical side of the theory of errors will be able to read the contents and use the results without having to read technical proofs of formulas, while those interested in their derivation can turn to the appendix and find this phase presented. The chapters are short and to the point.

The first two chapters present an introduction, two examples from experience showing how distributions similar to the normal curve arise, and a short discussion of probability.

Chapters 3 and 4 present the normal curve of error for linear, area, and volume distributions, together with formulas for h, the standard deviation, and mean error. The probable errors for these three distributions are defined, discussed briefly, and given in terms of the standard deviation and mean error. The percentage error curves are introduced for these distributions with examples showing their construction and uses.

Chapter 5 is devoted to a short treatment of determining the curve of best fit and the most probable point by using the method of least squares. Exponential equations for predicting are treated linearly by using logarithms.

The following three chapters present the correlation coefficient, partial correlation, quality correlation, and the correlation ratios, together with a table exhibiting the probability of the correlation coefficient. A contingency table is explained for determining this coefficient.

The last chapter treats of the error of the last figure, the error of the interval, and the errors of functions. A power series raised to an integer is used to advantage in the discussion of the error of the interval.

The appendix follows with mathematical derivations of all formulas, equations of curves of reference, and the evaluation of certain integrals. A summary of formulas follows.

Many well constructed diagrams appear in the book and should prove helpful to the reader. Several chapters close with exercises for the student. The book is small, well bound, and easy to carry. It should prove to be very useful to those doing research work in this field. I found the treatise interesting and worthwhile.

W. D. BATEN

The Electromagnetic Field. By Max Mason and Warren Weaver. The University of Chicago Press, 1929. xiii+390 pp.

The object of this book is to give, as far as possible, a mathematically satisfactory treatment of the fundamental facts of electromagnetism. It is divided into four chapters entitled, respectively, Coulomb's Law and Some Analytic Consequences, The Electrostatic Problem for Conductors and Dielectrics, Magnetostatics, The Maxwell Field Equations, which are followed by a mathematical appendix. Each chapter is divided into several parts (each of which is preceded by an introduction telling briefly its object) and is followed by a general conclusion summarizing the results of the chapter. Great attention has been given to the mode of presentation and every serious student of the subject will do well to study the book.

As may be gathered from the title of the first chapter Coulomb's action-at-a-distance formula constitutes the foundation on which the discussion is based so that, philosophically, the authors have not the view-point of Faraday and Maxwell whose efforts were devoted to a replacement of the action-at-a-distance concept by the action-through-a-medium idea. The writer of this review finds himself a follower of Faraday and while admitting the necessity and value of a careful study (such as that given in this book) of the consequences of Coulomb's Law, especially in its statistical aspects, cannot recognize its physical significance. Geometry itself has witnessed the struggle between these two opposing points of view. Euclid was the great proponent of the action-at-a-distance concept while Riemann, actuated precisely by a desire to explain Faraday's results, initiated the study of the infinitesimal action-through-a-medium geometries which has led to the brilliant "connection" theories of Levi-Civita, Weyl, Cartan, and others.

Any endorsement of details on the part of the present reviewer would be presumptuous and so we shall not be misunderstood if we confine ourselves to mentioning the few points where we found it difficult to follow. The essential difficulty in the definition of electric intensity is, as the authors carefully point out, that the introduction of a test-charge modifies the field. The fact that the internal electrostatic energy Ψ_ϵ, of §4 of the book, depends on the position of the test-charge ϵ seems to invalidate the statement that "the portion ψ_ϵ contains all the terms of Ψ which depend upon x_ϵ, y_ϵ, z_ϵ, the co-ordinates of the charge." If this is so the authors are not freed from their own criticism of "basing physical theories on definitions which involve what would happen if an impossible thing were true" (p. 72). The only other point we shall mention is mathematical. The authors first find the expression for the divergence of the electric intensity and then define the electric induction (or displacement) vector in terms of electric intensity and polarization. Now the divergence of a vector is a mathematical concept which derives from Stokes' Theorem on the flux of a vector-field through a closed surface. The electric intensity vector is never integrated through a surface but always along a curve (yielding electric potential) and hence the divergence of electric intensity has no real physical meaning while the divergence of the electric induction vector has, since the characteristic property of this vector-field is that its integral over any closed surface gives the charge enclosed by the surface.

F. D. MURNAGHAN

NOTES

Two-volumes of a new Roumanian mathematical journal, Mathematica, have appeared, under the editorship of Professors G. Tzitzeica and D. Pompieu, of Bucharest. Professor P. Sergescu, of Cluj, is secretary of the editorial committee.

The Ninth International Actuarial Congress was held at Stockholm, June 16–20, 1930, under the presidency of Director Sven Palme and Professor E. Phragmen. Four hundred and eighty delegates attended.

The Italian Society of Sciences has awarded its prize in mathematics for the year 1929 to Professor L. Fantappiè, of the University of Palermo, for his researches on analytic functionals.

The Rumford Premium of the American Academy of Arts and Sciences has been awarded to Dr. J. S. Plaskett, director of the Dominion Astrophysical Observatory, Victoria, British Columbia, for his work in stellar spectroscopy.

The University of Freiburg has conferred the honorary degree of doctor of natural philosophy on Dr. F. W. Aston, of Cambridge, for his work on isotopes and other subjects.

Sir William Bragg, director of the Royal Institution and of the Davy-Faraday Research Laboratory, has been elected corresponding member of the Vienna Academy of Sciences.

Cambridge University has conferred honorary doctorates on Professors Albert Einstein and Max Planck, of the University of Berlin.

Professor A. S. Eddington has been raised to knighthood. He has also been elected British honorary fellow of the Royal Society of Edinburgh.

The honorary degree of doctor of science has been conferred on Sir James Hopwood Jeans, secretary of the Royal Society of London, by the University of Dublin.

Professor Tullio Levi-Civita, of the University of Rome, has been elected a foreign member of the Royal Society of London.

Professor G. Vitali, of the University of Padua, has been elected a corresponding member of the Royal Academy dei Lincei.

The University of California has conferred an honorary doctorate on Professor Florian Cajori.

Harvard University has conferred an honorary doctorate on K. T. Compton, newly elected president of the Massachusetts Institute of Technology.

Indiana University has conferred honorary doctorates on Dr. C. O. Lampland and Dr. V. M. Slipher, of the Lowell Observatory, Flagstaff.

Lake Forest College has conferred an honorary doctorate on Professor W. D. MacMillan, of the University of Chicago.

Professors P. Franklin and D. J. Struik, of the Massachusetts Institute of Technology, have been elected fellows of the American Academy of Arts and Science.

A portrait, by Mr. Ralph Clarkson, of Professor E. H. Moore, of the department of mathematics of the University of Chicago, has been presented

to the University by Professor Moore's former students. It will be hung in the new Eckhart Hall of Mathematics, Physics and Astronomy.

Dr. W. F. Willcox, of Cornell, will represent the United States at the meeting of the International Institute of Statistics which will be held at Tokyo in September, 1930.

Professor H. Brandt, of the Aachen Technical School, has been appointed professor of mathematics at the University of Halle.

Mr. W. M. Smart, of Trinity College, has been appointed University lecturer in mathematics at Cambridge.

Professor G. P. Thomson, of the University of Aberdeen, has been appointed professor of physics at the University of London (Imperial College, Royal College of Science).

Mr. K. Yanagihara has been promoted to a professorship of mathematics at the Yamagata High School, Yamagata, Japan.

Dr. V. W. Adkisson has been appointed assistant professor of mathematics at the University of Arkansas.

Mr. J. G. Adshead has been appointed assistant professor of mathematics at Dalhousie University.

Assistant Professor J. P. Ballantine, of the University of Washington, has been promoted to an associate professorship of mathematics.

Dr. M. A. Basoco, of the California Institute of Technology, has been appointed assistant professor of mathematics at the University of Nebraska.

Dr. P. M. Batchelder has been promoted to an associate professorship of mathematics at the University of Texas.

Dr. Theodore Bennett has been appointed assistant professor of mathematics at the University of Wisconsin.

Assistant Professor F. E. Carr, of Oberlin College, has been promoted to a professorship of mathematics.

Dr. R. V. Churchill has been promoted to an assistant professorship of mathematics at the University of Michigan.

Associate Professor C. W. Cobb, of Amherst College, has been promoted to a professorship of mathematics.

Assistant Professor A. J. Cook has been promoted to an associate professorship of mathematics at the University of Alberta.

Mr. Wayne Dancer has been appointed assistant professor of mathematics at Toledo University.

Assistant Professor P. H. Daus, of the University of California at Los Angeles, has been promoted to an associate professorship of mathematics.

Dr. H. A. Davis, of West Virginia University, has been promoted to an assistant professorship of mathematics.

Assistant Professor H. T. Davis has been promoted to an associate professorship of mathematics at Indiana University.

Assistant Professor W. H. Durfee, of Hobart College, has been promoted to a professorship of mathematics.

Dr. C. M. Erikson, of Michigan State Normal College, has been promoted to an assistant professorship of mathematics.

Dr. H. P. Evans has been promoted to an assistant professorship of mathematics at the University of Wisconsin.

Mr. J. H. Fithian has been promoted to an assistant professorship of mathematics at the Newark College of Engineering.

Assistant Professor Philip Franklin, of the Massachusetts Institute of Technology, has been promoted to an associate professorship of mathematics.

Mr. H. J. Gay has been promoted to an assistant professorship of mathematics at the Worcester Polytechnic Institute.

Professor J. W. Glover, of the University of Michigan, has been appointed president of the Teachers Insurance and Annuity Association of America.

Assistant Professor Cornelius Gouwens, of Iowa State College, has been promoted to an associate professorship of mathematics.

Assistant Professor W. C. Graustein, of Harvard University, has been promoted to an associate professorship of mathematics.

Dr. Marion C. Gray has been appointed to a position in the department of development and research of the American Telephone and Telegraph Company.

Dr. Lois W. Griffiths, of Northwestern University, has been promoted to an assistant professorship of mathematics.

Dr. E. K. Haviland has been appointed assistant professor of mathematics at Lehigh University.

Dr. G. A. Hedlund has been appointed associate in mathematics at Bryn Mawr College.

Assistant Professor Einar Hille, of Princeton University, has been promoted to an associate professorship of mathematics.

Dr. J. J. L. Hinrichsen has been appointed assistant professor of mathematics at Iowa State College.

Dr. H. M. Hosford has been appointed to a professorship at the University of Arkansas.

Assistant Professor C. M. Huber, of Rutgers University, has been appointed professor and head of the department of mathematics at Atlantic University, Virginia Beach.

Assistant Professor Emma Hyde has been promoted to an associate professorship of mathematics at Kansas State Agricultural College.

Miss Myra I. Johnson has been appointed head of the department of biological science at Whitney Women's College.

Associate Professor R. A. Johnson, of Hunter College, has been appointed associate professor of mathematics at Brooklyn College of the City of New York.

Dr. B. W. Jones has been appointed assistant professor of mathematics at Cornell.

Dr. F. W. Kokomoor, of the University of Florida, has been promoted to an associate professorship of mathematics.

Dr. A. Marguerite Lehr, of Bryn Mawr College, has been promoted to be associate in mathematics.

Assistant Professor V. F. Lenzen has been promoted to an associate professorship of physics at the University of California, Berkeley.

Mr. A. J. Lewis has been promoted to an assistant professorship at the University of Denver.

Assistant Professor Mayme I. Logsdon has been promoted to an associate professorship of mathematics at the University of Chicago.

Professor Gertrude I. McCain, of Westminster College, has been appointed professor of mathematics at State College, Radford, Va.

Assistant Professor C. C. MacDuffee, of Ohio State University, has been promoted to an associate professorship of mathematics.

Assistant Professor H. F. MacNeish, of the College of the City of New York, has been promoted to an associate professorship of mathematics.

Assistant Professor E. B. Mode has been promoted to a professorship of mathematics at Boston University.

Dr. W. L. Moore has been promoted to an assistant professorship of mathematics at the University of Louisville.

Dr. C. O. Oakley has been promoted to an assistant professorship of mathematics at Brown University.

Assistant Professor H. B. Phillips, of the Massachusetts Institute of Technology, has been promoted to an associate professorship of mathematics. Professor Phillips has been granted leave of absence for the academic year 1930–1931.

Dr. C. G. Phipps has been promoted to an associate professorship of mathematics at the University of Florida.

Assistant Professor G. Y. Rainich, of the University of Michigan, has been promoted to an associate professorship of mathematics.

Assistant Professor C. H. Rawlins, of the United States Naval Academy, has been promoted to an associate professorship of mathematics.

Associate Professor J. F. Reilly, of the State University of Iowa, has been promoted to a professorship of mathematics.

Assistant Professor H. A. Ruger has been promoted to an associate professorship of educational psychology at Teachers College, Columbia University.

Assistant Professor C. A. Rupp has been promoted to an associate professorship at Pennsylvania State College.

Dr. Edward Saibel has been appointed assistant professor of mechanics at the Carnegie Institute of Technology.

Assistant Professor H. M. Sheffer has been promoted to an associate professorship of philosophy at Harvard University.

Dr. J. A. Shohat has been appointed lecturer in mathematics at the University of Pennsylvania.

Assistant Professor S. P. Shugert has been promoted to a professorship of mathematics at the University of Pennsylvania.

Professor C. D. Smith, of Louisiana College, has been appointed professor and head of the department of mathematics at the Mississippi Agricultural and Mechanical College.

Dr. R. G. Smith has been appointed associate professor of mathematics at Kansas State Teachers College, Pittsburg.

Assistant Professor E. B. Starke, of Rutgers University, has been promoted to an associate professorship of mathematics.

Dr. C. W. Strom has been promoted to a professorship of mathematics at Luther College.

Assistant Professor D. J. Struik has been promoted to an associate professorship of mathematics at the Massachusetts Institute of Technology.

Professor J. L. Synge, of Trinity College, Dublin, has been appointed professor of applied mathematics at the University of Toronto.

Professor R. A. Thornton, of Kittrell College, has been appointed head of the department of mathematical sciences at Talladega College.

Associate Professor F. C. Touton, of the University of Southern California, has been promoted to be professor of educational research and vice-president of the University.

Dr. H. S. Wall, of Northwestern University, has been promoted to an assistant professorship of mathematics.

Dr. Morgan Ward has been promoted to an assistant professorship of mathematics at the California Institute of Technology.

Assistant Professor F. M. Weida, of Lehigh University, has been appointed associate professor of mathematics at George Washington University.

Dr. Marie J. Weiss has been appointed assistant professor of mathematics at Sophie Newcomb College, Tulane University.

Assistant Professor W. M. Whyburn has been promoted to an associate professorship of mathematics at the University of California at Los Angeles.

Mr. F. H. M. Williams, of Drexel Institute, has been promoted to an assistant professorship of mathematics.

Dr. B. C. Wong has been promoted to an assistant professorship of mathematics at the University of California, Berkeley.

Associate Professor W. A. Zehring, of Purdue University, has been promoted to a professorship of mathematics.

The following appointments to instructorships are announced:
Brooklyn College of the City of New York: Mr. A. W. Landers, Jr.;
Case School of Applied Science: Dr. O. E. Brown;
Lehigh University: Dr. E. H. Cutler;
Los Angeles Junior College: Mr. E. J. Hills;
State Teachers College, Oshkosh: Miss Irene Price;
Princeton University: Mr. A. L. Foster, Mr. J. E. Merrill;
Purdue University: Mr. H. F. S. Jonah;
Radcliffe College: Dr. Mary Curtis Graustein (tutor);
Rutgers University: Mr. H. S. Grant;
Sweet Briar College: Dr. Ethel I. Moody;
United States Naval Academy: Mr. Ernest Hawkins.

Professor W. H. Bristol, inventor, and formerly professor of mathematics at Stevens Institute of Technology, has died at the age of seventy.

Professor Florian Cajori, of the University of Calfornia, well known for his books and writings on the history of mathematics, died suddenly on August 14, 1930, at his home in Berkeley. He had been a member of the American Mathematical Society since 1899. It is hoped that a more adequate statement regarding his life and work may be ready for publication in an early issue of this Bulletin.

Mr. E. A. Sperry, inventor of the Sperry gyro-compass and other gyroscopic appliances, died June 16, 1930, at the age of sixty-nine.

Dr. J. M. Taylor, professor emeritus of mathematics at Colgate University, died July 31, 1930, at the age of eighty-six.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume.* Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

317. Professor C. N. Moore: *On certain equivalent methods of summation.*

Given a series Σa_n and a positive monotonic function of n, λ_n, which becomes infinite with n. If $\lim_{n\to\infty}\Sigma_{m=1}^{n-1}(1-[\lambda_m/\lambda_n])^k a_m$ exists and is equal to A, we say that the series is summable (R,λ_n,k) to the value A. If $\lim_{\omega\to\infty}\Sigma_{m=1}^{\lambda_m<\omega}(1-[\lambda_m/\omega])^k a_m$ exists and is equal to A, we say that the series is summable (R,λ,k) to A. Both methods of summation are due to Marcel Riesz; the latter form is more generally used. It has been shown (Comptes Rendus, vol. 152 (1911), p. 1651) by Riesz that summability (R,λ,k), where $\lambda_n=n$, is equivalent to summability (C,k) for all real values of $k>-1$, and later (Proceedings of the London Mathematical Society, (2), vol. 22 (1923–24), p. 418) that summability (R,λ_n,k) for the same choice of λ is equivalent to summability (C,k) for $-1<k\leqq 1$. In the present paper it is shown that summability (R,λ_n,k) for the case $\lambda_n=c(n+q+[\psi_n/n])$, where ψ_n represents a function of n that remains bounded for all values of n and c and q are constants $(c>0)$, is equivalent to summability (R,λ_n,k) for $\lambda_n=n$, provided $-1<k\leqq 1$. In view of Riesz's second result, quoted above, this also involves the equivalence of the former type of summability with summability (C,k) if $-1<k\leqq 1$. (Received August 1, 1930.)

318. Professors Einar Hille and J. D. Tamarkin: *On the summability of Fourier series.* Third note.

The authors have continued their investigation of the effectiveness of the methods $[H,q(u)]$ of Hurwitz-Silverman-Hausdorff when applied to the summation of Fourier series. A distinction is made between F (Fejér) and L (Lebesgue) effective methods according as the applicability is ensured merely at regular points or almost everywhere. The authors exhibit sufficient conditions, in part also necessary, for both kinds of effectiveness. These conditions affect the integrability properties of the Fourier cosine transform of $q(u)$, and the continuity of $q(u)$ at the end points of the basic interval (0,1). The paper also contains a study of the integrability properties of Fourier transforms. (Received August 7, 1930.)

* See pp. 1–2 and p. 45 of the January issue.

319. Professor Dunham Jackson: *Note on the convergence of a sequence of approximating polynomials.*

Various results have been published with regard to the convergence of sequences of approximating polynomials defined by minimizing an integral which contains the mth power of the absolute value of the error, with a non-negative weight function. This note points out that the restrictions as to the vanishing of the weight function can be materially lightened, in comparison with the results cited, if it is assumed that the function to be approximated is analytic over a sufficiently extended region of the complex plane. (Received August 1, 1930.)

320. Professor R. L. Jeffery: *An application and an extension of Pierpont's theory of integration.*

In a short paper (Proceedings of the London Mathematical Society, (2), vol. 31, No. 1) we have discussed a problem stated in terms of Lebesgue's theory of integration for the solution of which this theory does not seem adequate. In the present paper we first show that the problem can easily be handled by means of Pierpont's theory. We then give a definition of an integral of a bounded function on a bounded set which is stated in terms of outer measure only, and show that this integral exists for an extensive class of non-measurable functions. (Received August 8, 1930.)

321. Professor F. L. Griffin: *As to a Goldschmidt solution for a wire of minimum attraction.*

In the problem of a wire exerting minimum attraction upon a particle at $O(0, 0)$, recently treated by Miss Swanson, no minimizing extremal can join $A(r_1, \theta_1)$ and $B(r_1, -\theta_1)$ in case $\theta_1 > 50°.24$, approximately. The present author points out that the analog of Goldschmidt's discontinuous solution for the minimal surface of revolution does not exist, though approximations thereto exist which give smaller attractions than certain combinations of extremal arcs in the case mentioned. (Received August 11, 1930.)

322. Professor T. R. Hollcroft: *Pencils of hypersurfaces.*

In a pencil of hypersurfaces in r-space, the number of hypersurfaces are found (1) that are tangent to a given manifold of any order and dimension; (2) that have a hypernode; (3) that have $r+1$ fixed points (or a hypernode at a fixed point) on a polar hypersurface; (4) whose Hessian passes through a fixed point; (5) that have contact with the Hessian. Involutions associated with the pencil and its polar hypersurfaces, the envelope of the Hessians of hypersurfaces of the pencil, the reduction in the number of hypernodes due to an s-fold basis point, are obtained. It is proved that the section of a general pencil of hypersurfaces in S_r by any linear space S_i, $i<r$, is a general pencil of hypersurfaces in S_i. (Received August 2, 1930.)

323. Dr. S. S. Cairns: *An axiomatic basis for euclidean plane geometry.*

Hilbert (*Grundlagen der Geometrie*, 1913, Anhang IV, pp. 163–218) has shown that three axioms pertaining to a set T of transformations carrying a plane into itself form a sufficient foundation for plane geometry. Various simplifications result from admitting orientation-reversing transformations and changing Hilbert's second and third axioms, as follows. Axiom 1: *The transformations T form a group.* Axiom 2: *Any two points, A and B, are invariant under some orientation-reversing transformation.* Axiom 3: *Any subgroup consisting of those transformations having two or more given fixed points in common is closed.* The points invariant under a transformation satisfying Axiom 2 constitute the *line AB*, and the transformation is called a *reflection.* As a consequence of the axioms, it is involutory. Hence two points uniquely determine a line. The images of a point B under all reflections leaving a point A fixed constitute a circle. Circles and lines are readily proved to be topologically equivalent to ordinary circles and lines in the (x, y)-plane. By some transformation, any line through a point A may be carried into any line through a point B in such a way that A goes into B. Other definitions and properties may readily be established. (Received August 11, 1930.)

324. Professor H. B. Curry: *Functionality in combinatory logic.*

The present paper is a development of combinatory logic (see abstracts 36–3–124 and 36–3–125) so as to apply to statements of the form "x belongs to the category α," where α is a proposition, function, Formel, Menge, or the like. An entity F (functionality) is introduced, such that the meaning of $(F\alpha\beta f)$, for instance, is "f is a function from α to β" or "fx belongs to β whenever x belongs to α," that of $(F\alpha(F\beta\gamma)f)$ is "fxy belongs to γ whenever x belongs to α and y to β," etc. What we may call the character of any entity is thus expressible by means of F and certain fundamental categories. Moreover an entity constructed by the processes already considered out of given entities whose characters are known will in general have a character dependent on the characters of the given entities. It is shown in this paper that any particular conclusion of this kind can be formally proved. The nature of the fundamental categories is irrelevant to this discussion and is not considered. However, a preliminary study of those paradoxes which are amenable to present treatment indicates that these all involve a fallacious assertion of the form under discussion. (Received August 7, 1930.)

325. Professor J. H. Neelley: *Concerning the possibility of a certain binary quartic being a line section of the plane quartic curve of genus zero.*

The nature of the roots of the catalectic cubic of the fundamental pencil have been established in this paper. The covariant quartics of the base quartics have been examined and one of them proves to be of special interest in that it can be a line section of the curve only if the curve osculates itself. Even in that case this covariant must be formed from a fundamental pencil with one catalectic base quartic. (Received August 11, 1930.)

326. Mr. A. L. Foster: *A continuous development of mathematical logic in finite terms.*

This paper develops the classical mathematical (formal) logic as a strictly finite discipline. Only finite sets are considered throughout, where intuitive clarity reigns. The usual axiomatic development, with all its multi-valued possibilities of expression (for example, *eq.* $\bar{p}\bar{v}\bar{q}$, etc.) is discarded for a single-valued one, a refinement of the truth-table. All propositional functions (of the various types) are determined in terms of this representation, and all the truth-functions; the complete classification of the latter is reduced to a purely group-theoretical one. Among these possible truth-functions appear the classical $(Ex)F(x)$ and $(x)F(x)$ ("exists" and "every") but also others, for example the definite existence $(Dx)F(x)$, which is foreign to classical logic but is the existence (and the only one) recognized by the intuitionistic school. The theory of implication as that of pure deduction is developed, in which it appears that $(Ex)F(x)$ is empty in that nothing (non-trivial) can be deduced from it. Also $(Dx)F(x)\rightarrow(Ex)F(x)$, but not the reverse. The continuity of this development, as opposed to the classical one in which (Ex), (x) are inserted axiomatically, is basically responsible for the uncovering of fundamental facts. The whole language of formal logic is built up as a continuous, self-consistent, deductive system, without ever passing from the finite. (Received August 2, 1930.)

327. Professor W. A. Wilson: *A property of continua similar to local connectivity.*

The definition of local connectivity about a point in a continuous space Z is readily generalized to local connectivity about a sub-continuum x of Z. Recently G. T. Whyburn has extended the notion of ϵ-separability by a finite set of points to ϵ-separability by a finite set of continua. A bounded sub-continuum x of Z is called biregular if for every sub-continuum y of $Z-x$, Z is the union of two continua H and K such that $H\cdot x=K\cdot y=0$. Various properties of biregularity are proved and in particular it is shown that in strength biregularity lies between local connectivity and ϵ-separability. (Received August 4, 1930.)

328. Professor W. A. Wilson: *Semi-metric spaces.*

Let Z be a set of elements such that for each pair x and y there are two non-negative numbers xy and yx called the distances from x to y and from y to x, respectively, and satisfying the following axioms: (I) $xy=0$ if and only if $x=y$; (II) $xz\leqq xy+yz$. A space of this kind is called semi-metric. It is shown that these spaces have a large number of properties closely analogous to those of ordinary metric spaces and that they include two common types of topological spaces as special cases. (Received August 4, 1930.)

329. Professor G. T. Whyburn: *Concerning continuous curves without local separating points.*

In this paper it is shown that *if K is any closed totally disconnected subset of*

a continuous curve M having no local separating point and p and q are any two points of K, then there exists in M an arc pq containing K. Let A denote the property of a continuous curve to contain, for each of its closed totally disconnected subsets K, an arc containing K; and let B denote the property of having no local separating point. According to the above theorem, property B implies property A. If each maximal cyclic curve of a continuous curve M has property B, then M has property A if and only if the cyclic elements of M form a simple cyclic chain. Clearly if a cyclically connected continuous curve is uni-coherent, it has property B. Thus the uni-coherent continuous curve M has property A if and only if the cyclic elements of M form a simple cyclic chain. Some necessary as well as the above sufficient conditions for property A are found. It is also demonstrated that every plane non-dense continuous curve having no local separting point is homeomorphic with the Sierpinski universal plane one-dimensional curve. (Received July 30, 1930.)

330. Professor G. T. Whyburn: *Concerning the subsets of regular curves.*

It is shown in this paper that every subset of a regular curve which is of dimension > 0 contains a non-degenerate connected set at points. As a consequence of this theorem it follows that all subsets of a regular curve (indeed, all regular point sets) may be divided into the following three mutually exclusive classes: (1) sets containing continua; (2) punctiform sets which contain non-degenerate connected sets; (3) zero-dimensional sets. It is also remarked that a more restrictive classification may be given for the regular sets K^n $(n = 1, 2, 3, \cdots)$ consisting of all points of order exactly n of any compact and metric continuum. The set K^3 can be proved to be zero-dimensional. (Received July 30, 1930.)

331. Mr. C. M. Cleveland: *On the existence of acyclic curves satisfying certain conditions with respect to a given continuous curve.*

The author establishes the following theorem: If, in the plane, M is a bounded continuous curve which contains no domain and K is a totally disconnected closed subset of M, there exists an acyclic continuous curve T containing K and such that (1) all the end points of T belong to K, (2) the point set $M \cdot T$ is totally disconnected and $M-T$ is connected. (Received July 21, 1930.)

332. Dr. L. H. Bunyan: *Concerning a certain integral equation.*

The integral equation $u(x, y)=\lambda\iint_D K(x, y; \xi, \eta)\ u(\xi, \eta)\ d\xi d\eta$, when the kernel $K(x, y; \xi, \eta)$ has a logarithmic discontinuity at the point $(x, y)=(\xi, \eta)$, is transformed into a system composed of an integro-differential equation and a boundary condition. It is shown that this system is equivalent to the given integral equation in the sense that any solution of either is also a solution of the other. The integro-differential equation, without the boundary condition, is found to admit a solution analytic in λ and uniformly bounded for each value

of the parameter λ which is sufficiently large in absolute value and located in a specified portion of the complex λ plane. (Received July 26, 1930.)

333. Dr. J. A. Shohat: *On certain differential equations.*

The classical orthogonal polynomials of Jacobi, Laguerre, and Hermite satisfy, it is known, a differential equation of the form (1) $Ay''+By'+Cy=0$, where A, B, and C are polynomials in x. The same holds for the orthogonal polynomials given by Heine (*Handbuch der Kugelfunktionen*, vol. 1, 1878), which it seems appropriate to call "elliptic polynomials," since they are closely related to $x=\sin \operatorname{am} \theta$, as the trigonometric polynomials are related to $x=\cos \theta$. In this paper the second particular solution of (1) is given in terms of the Tchebycheff function of the second kind, and also a differential equation is set up for the latter function. (Received August 5, 1930.)

334. Professor H. T. Davis: *The inversion of a differential operator of infinite order of Fuchsian type.*

This paper solves the following equation: (1) $A_0(x)u+A_1(x)xu'+A_2(x)\cdot x^2u''/2!+\cdots=f(x)$, where the $A_i(x)$ are analytic about $x=0$. The theories of the Volterra integral equation on the one hand and q-difference equations on the other are formally included in this generalization. The main results are as follows. A transformation exists and is explicitly given between the coefficients of $A_i(x)$ and the coefficients of a generalized Euler differential equation of infinite order such that if $v(t)$ is a solution of the latter, then $u(x)$, given by the inversion of the Laplace integral equation $v(t)=\int_L e^{xt}u(x)dx$, where L is a properly defined path, is a solution of the homogeneous case of (1). The solution of the hypergeometric equation about the origin and the solutions of the q-difference equation $u(qx)=(1-x)u(x)$ about $x=0$ and $x=\infty$ are found in this manner. The non-homogeneous equation is solved by a second transformation between the coefficients of (1) and a generalized Euler equation such that if $Y(x, z)$ is the resolvent generatrix of the latter then the resolvent of (1) is given by $X(x, z)=\Sigma_{n=0}^{\infty}Y^{(n)}(0, x)z^n/n!$. (Received August 4, 1930.)

335. Professor Marston Morse: *The problems of Lagrange and Mayer under general end conditions.*

A general formulation of problems of the above mentioned type has been given by Bolza, who obtained necessary conditions analogous to the Euler equations and the transversality conditions. Bliss lightened Bolza's hypotheses, and gave a new form to the problem. In its simpler aspects in the plane, it was recognized by Hilbert and others that the conditions analogous to the Jacobi conjugate point condition could be given in terms of characteristic roots of an auxiliary boundary value problem. A necessary condition of the latter sort for the general problem has recently been obtained by Cope. *Sufficient conditions in this general problem have never been established; the present paper establishes such conditions.* For an abstract of some of the results obtained, see Proceedings of the National Academy, vol. 16 (1930), p. 229, where references may also be found. (Received August 4, 1930.)

336. Professor Norbert Wiener: *Prime number theory and Planck's law.*

The problem of inverting Planck's law so as to obtain the distribution in temperature of a mixed black body radiation when its distribution in frequencies is known involves the study of the Riemann zeta function. In particular, the asymptotic study of this radiation for low frequencies, and consequently for low temperatures, involves identically the same mathematics as is needed to prove the prime number theorem. The Lambert series furnishes a link between the two. (Received July 24, 1930.)

337. Professor A. H. Copeland: *A matrix theory of measurement.*

The matrix theory of measurement furnishes a means of testing the consistency of the assumptions of the theory of measurement. It is a generalization of the theory of admissible numbers. In fact an admissible number may be considered a matrix each term of which is one or zero. We shall be concerned with infinite matrices whose terms are arranged in sequence. Such matrices will be called variates. If a variate satisfies certain assumptions of the theory of measurement, it is called admissible. With every admissible variate is associated an accumulative probability function. The mean value of a variate is defined as the limit, as n becomes infinite, of the average of its first n terms. An admissible variate possesses the property that the mean value of any continuous function of it is the Stieltjes integral of that function taken with respect to the accumulative probability function associated with it. Furthermore, the terms of an admissible variate must be independent. It is proved that corresponding to any denumerable set of accumulative probability functions there exists a set of independent admissible variates. These facts, although far from obvious, have been assumed in the theory of measurement. (Received August 9, 1930.)

338. Professor W. H. Durfee: *Summation factors which are powers of a complex variable.*

This paper deals chiefly with series of the form $y(z)=\Sigma a_n z^{f(n)}$, where Σa_n is summable with the value s by some Cesàro mean, z is a complex variable, and $f(n)$ belongs to the class of logarithmico-exponential functions. The theorems set forth conditions on $f(n)$ under which $y(z)\to s$ as $z\to+1$ from within the unit circle. These conditions differ according as the rate of increase of $f(n)$ is greater or less than that of $\log n$; in the latter case additional restrictions, including C_1-summability, are imposed on a_n. The latter part of the paper discusses certain series of the type $y(x)=\Sigma(-1)^n x^{f(n)}$ which, owing to too rapid a rate of increase of $f(n)$, do not approach a limit, but oscillate as $x\to+1$ from below. In this connection a generalization is given of a theorem on power series due to Belinfante (Koninklijke Akademie van Wetenschappen, Verslag, vol. 32(1923), p. 472). (Received August 11, 1930.)

339. Dr. F. W. Perkins: *Some properties of the oscillation of harmonic functions.*

In two earlier papers (*On the oscillation of harmonic functions*, Annals of Mathematics, (2), vol. 27(1925), pp. 159–170; *An intrinsic treatment of Poisson's integral*, American Journal of Mathematics, vol. 50(1928), pp. 389–414) the author has studied certain properties of the oscillation of harmonic functions, the discussion being directed primarily to the treatment of problems in the plane. The present paper gives the results of further study of this subject, with particular attention to problems in space. (Received August 11, 1930.)

340. Professor H. T. Davis: *The generalized Euler differential equation of infinite order.*

This paper gives the solution of both the homogeneous and non-homogeneous cases of the following differential equation of infinite order: $a_0u+(a_1+b_1x)u'+(a_2+b_2x+c_2x^2/2!)u''+\cdots=f(x)$, which includes the Euler differential equation as a special case. The essential contribution of this paper is the attainment of the general inverse operator for the non-homogeneous equation and the explicit solution of the homogeneous equation in the form $u(x)=\Sigma_{r=0}^{r'} x^{\lambda+m_r}\log^{r-r}x(\phi_{r1}+\phi_{r2}/x+\phi_{r3}/x^2+\cdots)$, where λ is a root of a factorial series which generalizes the indicial equation of the differential equation of finite order with a regular singular point at the origin. When applied to the hypergeometric equation the theory of this paper gives the solution about the point $x=\infty$. In general no solution will exist about the origin, and an example is given of an equation with a solution about infinity for which the unit circle forms a natural bound. In illustration of the use of the inverse operator the Abel integral equation is solved by means of it. (Received August 4, 1930.)

341. Dr. Morris Marden: *On Stieltjes' differential equation.*

The differential equation under consideration is (1): $A(z)\,(d^2w/dz^2)+B(z)\cdot(dw/dz)+C(z)w=0$, where $A(z)$, $B(z)$, and $C(z)$ are polynomials of degrees p, $p-1$, $p-2$, and where $B(z)/A(z)=\Sigma_{j=1}^{p}\alpha_j/(z-a_j)$. Pólya has proved that, when the α_j are real positive numbers, the zeros of every polynomial solution of (1) lie in any convex region enclosing the points a_j. In its first part the present note shows that in this same convex region lie also the zeros of every polynomial $C(z)$ for which equation (1) has a polynomial solution. In its second part the present note generalizes Pólya's theorem to the case that the α_j are any complex numbers with positive real parts. (Received August 11, 1930.)

342. Professor C. O. Oakley: *Note on the methods of Sturm.*

The author considers the linear homogeneous differential equation of the second order in self-adjoint form. The point of view is that, simply by means of the Picone identity, oscillation and comparison theorems for the zeros of derivatives of solutions are directly developable. The theorems as stated apply both to zeros of derivatives of solutions and to zeros of solutions. (Received August 11, 1930.)

343. Professor J. L. Walsh: *Note on the overconvergence of sequences of polynomials of best approximation.*

This note is a continuation of some previous work by the present author. In notation previously used (Proceedings of the National Academy, vol. 16(1930), p. 297), the following result is proved. If the function $f(z)$ is analytic interior to C_R, then the sequence of polynomials of best approximation to $f(z)$ on C converges to $f(z)$ interior to C_R, uniformly on an arbitrary closed point set interior to C_R, (1) when C is an arbitrary limited rectifiable Jordan arc and best approximation is measured in the sense of least pth powers by a line integral along C, (2) when C is an arbitrary limited point set whose complement D (with respect to the entire plane) is simply connected and best approximation is measured in the sense of least pth powers by integration on the unit circle γ when D is mapped onto the exterior of γ so that the points at infinity correspond to each other. (Received July 17, 1930.)

344. Professor C. T. Holmes: *The approximation of harmonic functions in three dimensions by harmonic polynomials.*

The object of this paper is the investigation of conditions under which a harmonic function in three dimensions can be uniformly approximated by a harmonic polynomial, or by a harmonic rational function. It is shown that, if the boundary of a closed point set R satisfies a condition analogous to Raynor's criterion for the possibility of the solution of the Dirichlet problem, then any function continuous in R and harmonic in the interior can be approximated uniformly on R by a harmonic polynomial plus a finite number of harmonic functions each rational except for a factor which is the reciprocal of a distance. If the boundary of R is also the complete boundary of an infinite region, the approximating function is a harmonic polynomial. Attention is paid also to the manner in which the solution of the Dirichlet problem varies with the boundary of the region. (Received August 2, 1930.)

345. Dr. G. A. Hedlund: *Poincaré's rotation number and Morse's type number.*

The first points conjugate to the points of a periodic extremal in an ordinary problem in the calculus of variations define a transformation which is continuous and advances points so that order is preserved. Thus there is a rotation number corresponding to a periodic extremal. The type number of a periodic extremal is directly related to the number of conjugate points and a relation between these numbers is to be expected. This is established in this paper by proof that the rotation number determines the type number of not only one period, but of any number of successive periods. Conversely, the rotation number is determined by the sequence of type numbers of an increasing number of successive periods. The first part of the paper treats the orientable case, where the type number has been defined and its properties studied by Morse. In the second part, where the case of a non-orientable surface is considered, it is necessary to develop the theory of the type number. (Received August 5, 1930.)

346. Dr. G. A. Hedlund: *Geodesics on a two-dimensional Riemannian manifold with periodic coefficients.*

A geodesic segment on a manifold is of Class A if it affords an absolute

minimum to all ordinary curves joining its end points and on the manifold. An unending geodesic is of Class A if any segment of it is of Class A. Given the positive definite differential form $ds^2=Edu^2+2Fdudv+Gdv^2$, where E, F, and G are periodic functions of class C^{iv} in u and v, it is proved that there exists a constant D, depending only on E, F, and G, such that, in the (u, v) plane, no Class A geodesic segment can wander a distance greater than D from the straight line joining its end points. With the aid of this theorem it is proved that there exists a Class A unending geodesic of the type of any given straight line in the (u, v) plane, and conversely, any one Class A unending geodesic is of the type of one straight line. A number of related theorems follow readily. The results apply to the geodesics on a surface homeomorphic with a torus. An example shows that the results of a similar nature do not hold in the case of more than two dimensions. (Received August 5, 1930.)

347. Dr. Lucien Féraud: *On Birkhoff's Pfaffian systems.*

This paper is a contribution to the study of the analytical and dynamical significance of the Pfaffian systems introduced by Birkhoff in his Colloquium Lectures. (Received July 26, 1930.)

348. Dr. S. B. Littauer (National Research Fellow): *On the symmetric form of the Fourier sine and cosine transforms.*

By the application of the Plancherel theory of Fourier transforms, the conventional definition of the sine and cosine transforms is so modified as to be distinguished only by a difference in a parameter. This form of definition lends itself to determining the asymptotic behavior of one of a pair of reciprocal functions so defined from the asymptotic behavior of the other. (Received August 8, 1930.)

349. Dr. R. P. Agnew (National Research Fellow): *The effects of general regular transformations on oscillations of sequences of functions.*

In a paper soon to appear in the Transactions of this Society, the author considers the behavior of continuous oscillation, continuous convergence, uniform oscillation, and uniform convergence of sequences of functions under regular transformations with triangular matrices. The present paper gives an extension of that investigation which employs general regular sequence-to-function transformations. Applications of regular transformations with square matrices, and of the Euler-Abel and Borel-Sannia transformations, are discussed. (Received July 22, 1930.)

350. Dr. N. H. McCoy (National Research Fellow): *Note on the existence of a positive function orthogonal to a given set of functions.*

A sufficient condition for the existence of a positive continuous function orthogonal to each member of a given set of functions $f_i(x)$, $i=1, 2, \cdots, m$, continuous and linearly independent on a closed interval X $(a\leqq x\leqq b)$, has been given by L. L. Dines (Annals of Mathematics, (2), vol. 28 (1926), pp. 393–395). The purpose of this note is to give two other sufficient conditions for

the existence of such a function. Let $F_i^{(k)}(x) = \int_a^x F_i^{(k-1)}(x)\,dx$ $(i, k = 1, 2, \cdots, m)$, where $F_i^{(0)}(x) = f_i(x)$, and denote by $(-1)^k D_k(x)$ the determinant obtained from the matrix $(F_i^{(j)}(x))$ $(i = 1, 2, \cdots, m; j = 0, 1, \cdots, m)$ by striking out the kth column. It is shown that if there exists a value of x on X for which the $D_k(x)$ are all different from zero and of the same sign, then the given set of functions admits a function with the desired property. The other condition involves the existence of $m+1$ distinct values of x satisfying certain requirements. (Received August 11, 1930.)

351. Professor H. S. White: *The construction of a headless and groupless triad system on* 31 *elements.*

Triad systems on 15 elements may have no head, that is, contain no complete and closed system of triads on 7 elements, and may still admit a group of transformations into themselves; or they may be both groupless and headless. Of the latter sort are, for example, the last three systems discovered by F. N. Cole, Nos. 34, 35, 36. For 31 elements the existence of groupless triad systems was demonstrated and millions of them, all incongruent, were constructed, but all of them had heads, closed systems on 15 elements. The present note modifies one of these, namely one based on Cole's No. 36, and proves that the resulting Δ_{31} is both groupless and headless. (Received August 8, 1930.)

352. Professor A. J. Kempner: *Residue systems for a prime modulus.*

Let the element a_{ij} in a square array of p rows and columns be the residue, modulo p, of j^i. This table shows a greater degree of symmetry than seems to have been noticed, and permits the derivation of relations between the residues of a^i, a fixed, and j^c, c fixed. We deal with relations between certain subsets of these residues, rather than with the individual residues. In the second part of the paper contact is established with the "completely reduced polynomials" investigated by the author in previous papers. In particular, a theory of factorization for such polynomials in one or more variables is established. The results are at present restricted to a prime modulus. (Received August 9, 1930.)

353. Dr. H. T. Engstrom (National Research Fellow): *On sequences defined by linear recurrence relations.*

Periodicity for a rational integral modulus m in a sequence of rational integers defined by a linear recurrence relation of kth order has been studied by Carmichael for moduli m whose prime divisors exceed k. In the present paper the author obtains a solution of the problem without restriction on m by use of the theory of ideals. The results obtained include those of Carmichael. (Received August 11, 1930.)

354. Professors C. L. E. Moore and Philip Franklin: *Algebraic Pfaffians.*

This paper is devoted to a study of the geometric interpretation of the vanishing of a special type of Pfaffian. The coefficients are polynomials in

the variables, which are thought of as Cartesian coordinates in space of three dimensions. At times, however, homogeneous coordinates are used, and some of the results are extended to n variables. When the polynomials are at most of the second degree, there is a certain complex of straight lines included among the integral curves. These complexes are studied, and made the basis of a classification of the possible types of Pfaffians under discussion. (Received July 23, 1930.)

355. Professor R. D. Carmichael: *Expansions of arithmetical functions in infinite series.*

The remarkable expansions of arithmetical functions obtained by Ramanujan in his notable memoir (Collected Papers, No. 21) on certain trigonometrical sums and their applications are contained as special cases of much more general expansions which have also other special cases of particular interest. It is the purpose of this paper to present these generalizations and to draw from the expansions some conclusions of importance obtained by means of a hitherto unnoticed fundamental property of the Ramanujan sums $c_q(n)$, namely, that expressed by the relations $\sum_{n=1}^{q\rho} c_q(n)c_\rho(n)=0$ if $\rho\neq q$, $\sum_{n=1}^{q} c_q^2(n)=q\phi(q)$. This property has led to the notion of orthogonal arithmetical functions analogous to the notion of orthogonal functions in analysis. The most characteristic results of the paper are those relating to the asymptotic forms of certain functions defined by aid of the divisors of a given integer. The functions involved here have several applications which we hope to present later and in particular an important application to the problem of the representation of integers as sums of squares. (Received August 4, 1930.)

356. Professor R. D. Carmichael: *On the representation of integers as sums of an even number of squares or of triangular numbers.*

Let s and n be positive integers, $r_{2s}(n)$ the number of ways of representing n as a sum of $2s$ squares, and $\gamma_q(n)=\cos[\frac{1}{2}\pi s(q-1)]\sum_\lambda\cos(2\pi\lambda n/q)-\sin[\frac{1}{2}\pi s(q-1)]\sum_\lambda(-1)^{(\lambda-1)/2}\sin(2\pi\lambda n/q)$, $q=1, 2, 3, \cdots$, where the sums are taken for the positive integers λ which are prime to q and do not exceed q. This paper proves the formula $(1/m)\sum_{n=1}^{m}\gamma_\rho(n)n^{1-s}r_{2s}(n)=\pi^s\epsilon_\rho(s)\phi(q)/[(s-1)!\rho^s]+O(1/m)$, $s>3$, where ρ is any positive integer, $\phi(\rho)$ is the Euler ϕ-function and $\epsilon_\rho(s)$ is 1, 0, or 2^s according as ρ is odd, $\rho\equiv 2 \bmod 4$, or $\rho\equiv 0 \bmod 4$. For $\rho=1$ this says that the function $n^{1-s}r_{2s}(n)$ is in the mean (on the average) equal to $\pi^s/(s-1)!$ if $s>3$. Similar results are obtained for the function $\sum_{r,s}(n)$ of Ramanujan (Collected Papers, p. 136) and for certain functions connected with the problem of representing integers as sums of triangular numbers. This paper is based on important results due to Ramanujan (loc. cit., pp. 136–137, 179–199) and on a certain hitherto unnoticed property of orthogonality of the Ramanujan functions $\gamma_q(n)$. (Received August 4, 1930.)

357. Professor R. D. Carmichael: *Concerning quasi-k-fold transitivity of permutation groups.*

Let G be a permutation group having the property that for every l such

that $1 \leqq l \leqq k$ it is true that when two sets are given of l letters each, taken from the letters on which G operates, then a permutation P exists in G which transforms one of these sets in some order into the other; then it will be said that G is quasi-k-fold transitive. It is clear that G is transitive in the ordinary sense. A general theorem concerning the order of such groups is proved. Infinite classes of singly transitive and doubly transitive groups are exhibited such that they are respectively quasi-2-fold and quasi-3-fold transitive. A single example is given of a three-fold transitive group which is quasi-4-fold transitive. These examples arise from the linear fractional group in the Galois field $GF[p^n]$. (Received August 4, 1930.)

358. Professor R. D. Carmichael: *On a question related to Waring's problem.*

Let D_m $(m>1)$ denote the multiplicative domain which is generated by 0, 1, the prime factors of m, and the νth powers of the primes of the form $mx+\alpha$ where α ranges over the integers less than m and prime to m, and ν is the exponent to which α belongs modulo m. For such a domain the following questions (suggested by Waring's problem) are proposed: What is the least integer $g(m)$ such that every positive integer is a sum of $g(m)$ integers belonging to D_m? What is the least integer $G(m)$ such that all but a finite number of positive integers are each expressible as a sum of $G(m)$ integers belonging to D_m? What is the least integer $\Gamma(m)$ such that "nearly all" positive integers are each expressible as a sum of $\Gamma(m)$ integers belonging to D_m? These questions seem difficult in the general case. The following theorems are proved: (I) $g(3)=G(3)=\Gamma(3)=2$; (II) $g(4)=G(4)=\Gamma(4)=2$; (III) $g(6)=G(6)=\Gamma(6)=2$; (IV) $g(8)=G(8)=\Gamma(8)=3$; (V) $g(12)=3$, $G(12)=2$ or 3, $\Gamma(12)=2$ or 3; (VI) $g(24)=G(24)=\Gamma(24)$. Some empirical results are indicated for other cases. Certain algebraic forms associated with particular domains D_m are also investigated. (Received August 4, 1930.)

359. Professor E. T. Browne: *The characteristic roots of a matrix.*

This paper is concerned with the problem of finding an upper limit to the roots of the characteristic equation of a matrix A, real or complex, of order n, a problem which has engaged the attention of Bendixson, Hirsch, Bromwich, and others. By a remarkably simple method the author establishes the following theorem: If $S_i(T_i)$ is the sum of the absolute values of the elements in the ith row (column) of a square matrix A, and if $S(T)$ is the greatest of the $S_i(T_i)$, the absolute value $|\lambda|$ of a characteristic root λ of A cannot exceed $(S+T)/2$. Then by invoking results due to Hirsch and Bromwich, upper limits to the real and imaginary parts of λ are found. (Received July 15, 1930.)

360. Professor E. T. Browne: *On the separation property of the roots of the secular equation.*

Let A be an Hermitian matrix of order n and I the unit matrix. If $L_i(\lambda)$ is the principal minor determinant of order i standing in the upper left hand corner of $A-\lambda I$, it is well known that the roots of $L_i(\lambda)=0$ are all real and in

general separate the roots of $L_{i+1}(\lambda)=0$. However, in case the former equation has a multiple root the sense in which this separation takes place is not exactly clear, since a root of multiplicity m of $L_i(\lambda)=0$ may be a root of multiplicity $m-1$, m, or even $m+1$ of $L_{i+1}(\lambda)=0$. In this paper a study is made of the separation property in the case where $L_i(\lambda)=0$ has multiple roots. (Received July 15, 1930.)

361. Professor C. O. Oakley: *Semi-linear integral equations.*

We shall write as the type-form of the equations treated in this paper $y(x)+\sigma g(x)\left|y(x)\right|=f(x)+\lambda\int_\alpha^\beta K(x,\xi)y(x)d\xi+\mu\int_\gamma^\delta L(x,\xi)\left|y(\xi)\right|d\xi$. Existence theorems are derived by the method of successive approximations for small values of the parameters σ, λ, μ and with suitable restrictions upon g, f, K, L. The analogy with the linear case which is included is apparent. Outside of small circles, however, this analogy breaks down, for it is found that under certain conditions there is no solution for $\left|\mu\right|>c$, where c is a known constant. Further theorems of restriction show that in certain cases the region of existence of solutions may even be non-circular. (Received August 11, 1930.)

362. Dr. J. J. Gergen (National Research Fellow): *Convergence and summability criteria for Fourier series.*

This paper gives first a generalization (Theorem I below) of the classical Lebesgue criterion for the convergence of a Fourier series. It then discusses the relation of this test to the six commonly recognized criteria and certain of their generalizations. It is found that in this form the Lebesgue test strictly includes these others, in particular, Hardy's and Littlewood's. Theorem I is then extended in two directions. A slightly more general convergence criterion is obtained, and also a criterion for Cesàro summability of arbitrary order. It is proved that in one of the alternative forms of Lebesgue's original theorem the continuity condition is redundant. Theorem I may be stated as follows. Suppose that $f(t)$ is integrable (Lebesgue) over $(0, 2\pi)$, and periodic with period 2π. Then, if at the point $t=t_0$ the Fourier series of $f(t)$ is summable, to sum s in some Cesàro manner, and if (1) $\lim_{\epsilon\to\infty}\lim\sup_{x\to+0}\int_{\frac{1}{\epsilon x}}^{\pi}\left\{\left|\phi(t+x)-\phi(t)\right|/t\right\}dt=0$, where $\phi(t)=\left\{f(t_0+t)+f(t_0-t)\right\}/2$, the series converges there, to sum s. Moreover, when (1) holds, it is necessary and sufficient for summability, to sum s, at $t=t_0$, that $\lim_{x\to+0}\left\{x^{-1}\int_0^x\phi(t)dt\right\}=s$. (Received August 11, 1930.)

363. Professor B. F. Kimball: *A useful formula in the theory of finite differences.*

A formula showing a relation between the rth difference $\Delta_r f(x)$ and the rth derivative $f^{(r)}(x+\frac{1}{2}r)$ of a function is demonstrated, and certain related theorems and applications are discussed. (Received August 11, 1930.)

364. Dr. Morris Marden: *A rule of signs involving certain orthogonal polynomials.*

In his Oeuvres, vol. 1, pp. 144–146, Laguerre gives a rule for determining the number N of real zeros greater than or equal to unity of a given real polynomial $F(x)$. Namely, express $F(x)$ in terms of the Legendre polynomials

$P_j(x)$: $F(x)=c_0P_0(x)+c_1P_1(x)+\cdots+c_nP_n(x)$, and calculate the number V of variations of sign in the sequence $c_0, c_1, \cdots, c_n$. Then $N \leqq V$. The present note gives a similar rule, wherein the $P_j(x)$ are a set of any Jacobi or generalized Laguerre polynomials. (Received August 11, 1930.)

365. Dr. Dorothy W. Weeks: *A study of the interference of polarized light by the method of coherency matrices.*

Wiener has developed the method of analyzing polarized light by coherency matrices, and applied it to the case of a single polarized beam of homogeneous light. This paper extends it to the case of two simultaneous beams of homogeneous light. The general form for this matrix is given, and sixteen fundamental coherency matrices are made the basis of study. The transformation matrices that change one coherency matrix into another are determined. Eight of the coherency matrices represent types of incoherent and eight of coherent light. The transformation matrices fall into two main classes, the β and ϕ classes. The β class has three main subclasses, each representing a closed group of thirty-two elements. In determining the β class, twenty-four possible forms in three sets of eight satisfy the imposed conditions. These twenty-four forms constitute a closed group, reducible to a four, a three, and a two group. The three group contains a form from each of the three sets of eight, and has therefore been selected. The forms of the four group are the forms of the Dirac matrices. Since the coherency matrices represent different types of light, the transformation matrices must characterize the optical instruments that change one type of light into another. Michelson's interferometer can be represented by such a matrix. (Received August 1, 1930.)

366. Professor R. E. Langer: *On the flow of heat for a solid in contact with a liquid.*

A cylindrical solid of length a in direction x is placed in contact at its face $x=a$ with a stirred liquid extending to $x=a+b$. The lateral surface is insulated. Heat may or may not be permitted to flow across the faces $x=0$, $x=a+b$. The initial temperatures in the solid and the initial temperature of the liquid being given, the subsequent temperatures are to be determined. The following points are of interest. (1) The method of Fourier leads to an expansion in non-orthogonal functions. This point has led previous investigators to an entire change of method (H. W. March and Warren Weaver, Physical Review, vol. 31 (1928), p. 1072) or to the use of special devices (R. L. Peek, Jr., Annals of Mathematics, vol. 30 (1929), p. 265). It is here shown to be amenable to systematic treatment. (2) If the initial temperature of the liquid differs from that of the solid the solution must be discontinuous at $t=0$. This point (ignored by some authors) explains physically a familiar pecularity of behavior of certain series expansions at the ends of the convergence interval. The problem can be phrased also as a problem in diffusion. (Received August 2, 1930.)

367. Professor K. W. Lamson: *Some differential and algebraic consequences of the Einstein field equations.*

If a set of four directions in a four-dimensional Riemannian space is ortho-

gonal, then the ds^2 can be expressed in terms of their sixteen parameters. The first purpose of this paper is to set up sixteen invariant linear first-order partial differential equations in these parameters. The solutions of these equations include all non-degenerate solutions of the Einstein field equations of 1917. The components of the curvature tensor may be taken as coefficients of the equation of a quadratic line complex in a three-dimensional space. The second purpose of the paper is the application of some of the theory of complexes to the theory of spaces satisfying the Einstein equations. (Received August 5, 1930.)

368. Mr. Nicholas Rashevsky: *Some new mathematical problems in biophysics.*

The theoretical study of certain physical phenomena which are supposed to occur in living organisms leads to various new types of mathematical problems, not hitherto studied in pure mathematics. As examples, two such problems are discussed. One relates to the determination of the shape of equilibrium of liquid drops, in which certain capillary active substances are constantly diffusing, causing a non-uniformity in the distribution of the surface tension along the surface. This problem has been sketched in a previous publication (Zeitschrift für Physik, vol. 56 (1929), p. 297) and is somewhat analogous to that of determining the unknown kernel of an integral equation from the known solution. Another still more unusual type of problem arises in connection with the theory of propagation of the nervous impulse. A few other problems are briefly mentioned. (Received August 11, 1930.)

369. Mr. B. F. Groat: *Mean value of the ordinate of the locus of the rational integral algebraic function of degree n expressed as a weighted mean of $n+1$ ordinates, and the resulting rules of quadrature.*

There are several well known rules, such as Newton's, Simpson's (or Cotes'), and others, for approximating to the areas of closed regions; and such approximations have been studied by Newton, Lagrange, Gauss, and others. The author of this paper does not know of any general formulas for finding the averaging weights of arbitrarily chosen ordinates which will give the true area under the general curve of rational integral algebraic form, nor of any rule for finding the mean ordinate of such a curve as the arithmetic mean of a determinate number of properly situated ordinates. In the present paper, the author derives rules and formulas for these purposes. (Received August 2, 1930.)

370. Dr. W. D. Baten: *Corrections for the moments of a frequency distribution in two variables.*

In certain statistical problems it is beneficial to divide the given data into classes or groups and then investigate the distribution in this form. The moments determined for the distribution divided into classes differ from the moments determined from the original data. It is the object of the present paper to show how to modify the former to secure the latter. The corrections

for the moments are obtained by use of the Euler-Maclaurin summation formula for two variables and certain assumptions concerning the frequency function at the limits of the distribution. (Received August 11, 1930.)

371. Professor H. W. Sibert: *Moderately thick circular plates with plane faces.*

The method of solution of the above mentioned problem used in this paper is based on the assumption that the displacements may be expanded in positive integral powers of z, the coefficients being polynomials in r. An infinite set of constants is developed which makes it possible to prove convergence for a limited class of loading functions. This is the first time that convergence has been proved for a method based upon expansion in positive integral powers of r and z. Nádai and Clemmow have each given solutions involving Bessel functions which are convergent for a limited class of loading functions, but the calculations involved are so difficult that it would be almost impossible to extend their methods to the more complicated types of loading conditions that can be treated by the method of this paper. A general formula for the displacements is obtained, from which a solution may be derived for any loading independent of θ which results in convergent series for the displacements. In applying this method to physical problems, it has been possible to obtain all the constants of integration by a rigorous mathematical treatment not dependent on physical intuition. (Received July 16, 1930.)

372. Miss Marian A. Wilder: *Correlation coefficients and transformation of axes.*

It is well known that coefficients of correlation can be interpreted as cosines of angles, either as a direct application of the fundamental formulas of analytic geometry in space of n dimensions, or, in the case of a normal frequency distribution, in connection with transformation of the axes of reference for the ellipsoids of equal frequency. This paper shows how a configuration corresponding to that which forms the basis of the latter interpretation, and reducing to it in the case of a normal distribution, can be defined for a frequency distribution of arbitrary form. (Received August 1, 1930.)

373. Dr. A. C. Berry (National Research Fellow): *Linear transformations in conjugate abstract spaces.*

M. Riesz (Acta Mathematica, vol. 49 (1926), pp. 465–497) has studied the convexity properties of the bounds of linear transformations operating on functions whose pth powers possess a Lebesgue-Stieltjes integral with respect to a given non-decreasing function. Conjugate exponents p, p' play an important rôle. In the present paper there are introduced general notions of convexity and of conjugate abstract spaces. The important theorems of Riesz are generalized and the proofs in some instances simplified. Following Riesz's suggestion the theory of the Fourier transform is given as an application and is thus established without reference to the theory of Fourier series. (Received August 16, 1930.)

374. Dr. Hillel Poritsky: *On conservative transformations connected with certain harmonic functions.*

By a conservative transformation of a surface into itself is meant (following Birkhoff) a transformation possessing a positive invariant surface integral. The importance of such transformations in dynamics has been shown by the work of Poincaré and Birkhoff. In this note attention is called to certain conservative transformations connected with harmonic functions of the form $v=\sum_{i=1}^{k} c_i/r_i$, where the c_i are constants and the r_i represent the distances from k fixed points, P_i, respectively. The transformation is that of a complete equipotential surface $v=v_0$ into itself; the transform $T(P)$ of a point P of $v=v_0$ is obtained by following along the line of force through P, first in the direction of, say, increasing v, through one of the points P_i or the point at infinity, and until the line of force intersects $v=v_0$ again; the latter point of intersection is $T(P)$. This transformation is one to one and continuous except possibly at the points which lie on lines of force passing through points of vanishing force, and admits as invariant integral the "flux" integral $\int(\partial u/\partial n)dS$. If the constants c_i are all positive and v_0 is small enough, $v=v_0$ is approximately spherical; when v_0 approaches 0, a transformation of a sphere into itself is obtained which preserves areas. (Received August 12, 1930.)

375. Professor W. H. McEwen: *Problems of closest approximation connected with the solution of linear differential equations.*

This paper is concerned with the approximate representation of a solution of a given linear ordinary differential equation of the mth order with boundary conditions, by means of linear combinations of given functions, particularly by means of trigonometric sums and polynomials, the approximating functions being subjected to the boundary conditions of the differential system. The criterion of approximation is the minimizing of the integral of $|L(y)-L(y_n)|^r$, where y is the function to be approximated, $L(y)$ the differential expression forming the left-hand member of the differential equation, and y_n the approximating function. Related problems have been discussed recently by Kryloff, Picone, and others, but the methods and results are different. The convergence proofs for $r>1$ are based on a simple and direct application of Hölder's inequality, and for $r<1$ on an extension of Bernstein's theorem on the derivative of a trigonometric sum or polynomial. (Received August 21, 1930.)

376. Professor O. D. Kellogg: *Capacity of sets of Cantor type.*

The fundamental lemma that every closed bounded set of positive capacity has a regular point has been proved for the logarithmic potential (Comptes Rendus, vol. 187 (1928), p. 526), but not as yet in space of three dimensions. The importance and difficulty of the problem make special cases of interest (see American Journal of Mathematics, vol. 51 (1929), pp. 515–526). Among sets for which the lemma might seem most likely to fail are those of Cantor type formed as follows. From the unit cube are removed all points whose distances from the planes through the center and parallel to the sides are less than $\alpha_1/2$, the remaining points constituting the set E_1. From the cubes of E_1 are removed all points whose distances from the corresponding planes are less than $\alpha_2/2$

times the length of the sides of the cubes, all the remaining cubes constituting the set E_2, and so on. The set E consisting of all the points common to the infinite sequence of sets $E_1, E_2, E_3, \cdots$ is the set of Cantor type corresponding to the sequence of positive proper fractions $\alpha_1, \alpha_2, \alpha_3, \cdots$. Such sets may have positive, or zero, capacity, according to the sequence of α's. But if this sequence is either bounded away from 0, or converges to 0, the fundamental lemma holds for the set. It is also shown that sets with positive outer Jordan content always contain regular points. (Received August 21, 1930.)

377. Professor H. S. Wall: *On the Padé approximants associated with certain divergent series.*

The following theorem applies to (a) Stieltjes series, (b) positive definite series, (c) certain normal series having corresponding continued fractions

$$\cfrac{1}{a_1 z + a_2 + \cfrac{1}{a_3 z + \cdots}}$$

in which $\sum |a_i|$ converges. Theorem: *Let the principal diagonal file, S_0, of the associated Padé table, and the first parallel file, S_{-1}, below S_0, converge and have different limits. Then an arbitrary file S_k converges to a limit of the form u_k/v_k, where $u_k = \alpha_k p - \beta_k p_1$, $v_k = \alpha_k q - \beta_k q_1$; p, q, p_1, q_1 are entire functions; α_k, β_k are polynomials in $1/z$; $\alpha_{-1} = 1$, $\beta_{-1} = 0$, $\alpha_0 = 0$, $\beta_0 = -1$; $u_k v_{k'} - u_{k'} v_k = \alpha_k \beta_{k'} - \alpha_{k'} \beta_k$.* Let $\bar{u}_k = u_k/(\alpha_k - \beta_k)$, $\bar{v}_k = v_k/(\alpha_k - \beta_k)$. Then if z is real, the points $(\bar{v}_k, \bar{u}_k)$, $k = 0, \pm 1, \pm 2, \cdots$, lie on a straight line, $L(z)$, which does not pass through the origin. For (a), (b), $L(x)$ rotates continuously clockwise about the origin as x increases from $-\infty$ to $+\infty$. (Received September 4, 1930.)

378. Dr. C. I. Lubin: *A further note on singular points of the differential equation $dx/X(x, y) = dy/Y(x, y)$.*

In the system $dx/dt = X(x, y)$, $dy/dt = Y(x, y)$, the real functions $X(x, y)$ and $Y(x, y)$ of the real variables x and y are supposed to be analytic in the neighborhood of the origin and to vanish at the origin. The matrix of the linear terms of the power series expansions of $X(x, y)$ and $Y(x, y)$ is assumed to be of rank 2 and to have characteristic roots $\pm\lambda(-1)^{1/2}$. This discussion is preliminary to further investigation regarding the nature of the functions accomplishing the transformation. The form attained by the formal reduction is $du/dt = v + \frac{1}{2}[Av - Bu](u^2+v^2)^m - \frac{1}{2}C(u^2+v^2)^{2m}u$, $dv/dt = -u - \frac{1}{2}[Au + Bv] \cdot (u^2+v^2)^m - \frac{1}{2}C(u^2+v^2)^{2m}v$, where if $B = C = 0$ the case of centers arises. This form can be further reduced to one with $A = 0$ by change of parameter. (Received September 5, 1930.)

379. Dr. W. J. Trjitzinsky (National Research Fellow): *A solution of a problem of Birkhoff.*

In 1913 Birkhoff stated, without proof, a theorem which involves the question of the possibility of constructing a function analytic in an open interval (0,1) and having the following properties: the function, together with its

derivatives of all orders, has assigned values at the end points of this interval; moreover, it is a sufficient approximation to a prescribed function. This problem has been solved by Besikowitsch (Mathematische Zeitschrift, 1924) by the aid of a non-uniform function. The present author gives a uniform solution by means of a series of rational fractions, utilizing for this purpose one of his earlier results. (Received September 8, 1930.)

380.* Dr. A. A. Albert: *On algebras of type R in thirty-six units.*

The algebras considered by L. E. Dickson on pp. 51–79 of his *Algebren und ihre Zahlentheorie* are called algebras of type R. The author considers here algebras D of type R and order thirty-six over any non-modular field, where D is based upon a non-cyclic sextic, with regular group, and proves that every such algebra is a cyclic (Dickson) algebra. (Received August 11, 1930.)

381. Professor R. G. Smith: *A canonical form for the differential equations of curves in n-dimensional space.*

In this Bulletin (vol. 34, pp. 290–302) Professor E. B. Stouffer derived canonical forms for the linear homogeneous differential equations associated with curves in a plane or in ordinary space, and showed their geometrical significance. The canonical form presented in this paper is a generalization of the canonical form for curves in ordinary space. The results obtained are valid for any space of more than two dimensions. It is shown that the vertices of the canonical polyhedron of reference are uniquely fixed with respect to the curve. In particular, the $(n+1)$th vertex is in the unique linear $(n-1)$-space determined by the osculating linear $(n-2)$-space and the principal tangent plane (a generalization of the principal tangent plane as used in ordinary space) of the curve and its osculating curve of degree n. The canonical form leads at once to a complete system of invariants and covariants. (Received August 5, 1930.)

382. Dr. P. M. Swingle: *Two types of connected sets.*

It is shown that a bounded plane indecomposable continuum contains a connected subset C, such that every connected subset of C is dense in C. A number of theorems are proved concerning such connected sets and a related type. (Received August 25, 1930.)

383. Professor R. D. Carmichael: *Algebras of certain doubly transitive groups.*

A class of finite algebras $A[p^n]$ is defined directly by means of doubly transitive groups of prime-power degree p^n and order $p^n(p^n-1)$ and is shown to be equivalent to a class of finite algebras defined by Dickson in 1905 (Göttinger Nachrichten, 1905). The set of all linear transformations on the

* The papers beyond this point are to be read at meetings of which the reports have not yet been published.

marks of an $A[p^n]$ induces on those marks a group which is conjugate to that by which the algebra is defined. Three forms are given to the (only partially solved) problem of determining all algebras $A[p^n]$, one of them being of fundamental importance in the investigation of the group of isomorphisms of an Abelian group of order p^n and type $(1,1,\cdots,1)$. This problem deserves further attention. Two algebras $A_1[p^n]$ and $A_2[p^n]$ are simply isomorphic when and only when their multiplicative groups are simply isomorphic. The integral elements of an algebra $A[p^n]$ form a Galois field $GF[p]$. The algebras $A[p^n]$ are capable of various analytical representations, including as a special case that employed by Dickson. A large class of doubly transitive groups of degree p^n and order $p^n(p^n-1)$ is exhibited and these groups are employed in the rapid construction of a large class of algebras $A[p^n]$ closely related to those determined by Dickson by other methods. (Received September 5, 1930.)

384. Professor R. D. Carmichael: *Tactical configurations of rank two.*

In accordance with the terminology of E. H. Moore, the term tactical configuration of rank two is employed to denote a combination of l elements into m sets, each set consisting of λ distinct elements and each element occurring in μ distinct sets (order of sets and order within a set being immaterial). Such configurations are important in the theory of permutation groups, in the formation of irrational invariants and in constructing poristic forms in connection with the study of geometrical configurations. In this paper numerous infinite classes of these tactical configurations are constructed by means of the finite geometries and by means of multiply transitive permutation groups. Several particular configurations of interest are also given. Means of constructing quadruple systems are set forth. The method of tactical configurations is employed in a new construction of the Mathieu groups of degrees 11, 12, 22, 23, 24. (Received September 5, 1930.)

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VOLUME I. **Linear Systems of Curves on Algebraic Surfaces,** by H. S. White; **Forms of Non-Euclidean Space,** by F. S. Woods; **Selected Topics in the Theory of Divergent Series and of Continued Fractions,** by E. B. Van Vleck. (Boston Colloquium.) New York, 1905. $2.75.

VOLUME II. **New Haven Colloquium,** by E. H. Moore, E. J. Wilczynski, and Max Mason. Out of print.

VOLUME III. **Fundamental Existence Theorems,** by G. A. Bliss; **Differential-Geometric Aspects of Dynamics,** by Edward Kasner. (Princeton Colloquium.) New York, 1913. $2.50.

VOLUME IV. **On Invariants and the Theory of Numbers,** by L. E. Dickson; **Topics in the Theory of Functions of Several Complex Variables,** by W. F. Osgood. (Madison Colloquium.) New York, 1914. $2.50.

VOLUME V, PART I. **Functionals and their Applications. Selected Topics including Integral Equations,** by G. C. Evans. New York, 1918. $2.00. PART II. **Analysis Situs,** by Oswald Veblen, out of print. (Cambridge Colloquium.)

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Mémorial des Sciences Mathématiques

Edited by HENRI VILLAT

Under the patronage of la Société mathématique de France, l'Académie des Sciences de Paris, and the scientific academies of Servia, Belgium, Bucarest, Poland, Ukraine, Spain, Prague, Rome (dei Lincei), and Sweden. See this BULLETIN, vol. 31, Nos. 5-6, p. 281.

In addition to twenty-five monographs that appeared before 1928, the following have recently been published.

26. B. GAMBIER, *Déformation des surfaces étudiées du point de vue infinitésimal.* 1928.
27. P. APPELL, *Le problème géométrique des déblais et remblais.* 1928.
28. E. COTTON, *Approximations successives et équations différentielles.* 1928.
29. C. GUICHARD, *Les courbes de l'espace à n dimensions.* 1928.
30. L. ZORETTI, *Les principes de la mécanique classique.* 1928.
31. B. GAMBIER, *Applicabilité des surfaces étudiée du point de vue fini.* 1928.
32. CH. RIQUIER, *La méthode des fonctions majorantes et les systèmes d'équations aux dérivées partielles.* 1928.
33. A. BUHL, *Aperçus modernes sur la théorie des groupes continus et finis.* 1928.
34. H. VERGNE, *Ondes liquides de gravité.* 1928.
35. LÉON LECORNU, *Théorie mathématique de l'élasticité.* 1929.
36. PAUL APPELL, *Sur la décomposition d'une fonction méromorphe en éléments simples.* 1929.
37. G. CERF, *Transformations de contact et problème de Pfaff.* 1929.
38. G. VALIRON, *Familles normales et quasi-normales de fonctions méromorphes.* 1929.
39. T. NAGELL, *L'analyse indéterminée de degré supérieur.* 1929.
40. S. LEFSCHETZ, *Géométrie sur les surfaces et les variétés algébriques.* 1929.

Many others have appeared or are in preparation.

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Bolza, Oskar: Lectures on the calculus of variations, Chicago U. Press, 1904.

Collectanea Mathematica in memoriam Dominici Chelini, Hoepli, 1881.

Memorie della reale accademia delle scienze di Torino, Ser. 2, Vol. 52, 1903.

Annali di Matematica: Ser. 1; Ser. 2, Vols. 1-4, 22, 24-26; Ser. 3, Vols. 14, 16.

Rendiconti del Circ. Matematico di Palermo, Vols. 35, 38, 40.

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Copies of this and the preceding monographs may be obtained by members of the Mathematical Association of America at cost through Secretary W. D. Cairns, Oberlin, Ohio. All other orders should be sent to the Open Court Publishing Company, Chicago, Ill. Price $1.25 each to members, $2.00 to others.

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 25, 1930.

Abstracts must be in the hands of the Secretary of the Society, Professor R. G. D. Richardson, 501 West 116th Street, New York City, not later than October 4. In order to be printed in the Bulletin in advance of the meeting, abstracts must be in the hands of the Secretary not later than September 7. By invitation of the program committee, Professor M. H. Stone will deliver an address entitled *Group representations in Hilbert space.*

COLUMBIA, MISSOURI, November 28-29, 1930.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, North Hall, Madison, Wisconsin, not later than November 6. All abstracts received by that date will appear in the Bulletin in advance of the meeting. On Saturday morning, Professor C. N. Moore will deliver an address by invitation of the program committee, on *Types of series and types of summability;* and Professor R. E. Langer will deliver an address on *The zeros of exponential sums and of certain related functions.*

UNIVERSITY OF CALIFORNIA AT LOS ANGELES, November 29, 1930.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, Berkeley, California, not later than November 1. All abstracts received by that date will appear in the Bulletin in advance of the meeting. At the request of the program committee, addresses will be delivered by Professor Harald Bohr on *The theory of Dirichlet series.*

CLEVELAND, OHIO, ANNUAL MEETING, December 29, 1930, to January 1, 1931.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, North Hall, Madison, Wisconsin, not later than December 1. In order to be printed in the Bulletin in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than November 7. There will be the usual joint session with Section A of the American Association for the Advancement of Science and with the Mathematical Association of America. There will be invited addresses by Professors G. D. Birkhoff, E. T. Bell, and others. A joint session with the American Statistical Society has been arranged for and it is probable that there will be a joint session with the American Physical Society. Professor E. B. Wilson will deliver the eighth Josiah Willard Gibbs Lecture.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

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WHOLE NUMBER 372

CONTENTS

	PAGE
The Thirty-Sixth Summer Meeting of the American Mathematical Society. By R. G. D. RICHARDSON	601
Dedekind's Works—Volume I. By E. T. BELL	611
Shorter Notices	613
Notes	623
Abstracts of Papers	628

For official Communications and Notices, see the inside of the back cover.